ACES-PHARAO test of the gravitational redshift : refined estimation of the expected uncertainty

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Outline

1. ACES-PHARAO test of the gravitational redshift
   - ACES-PHARAO mission
   - Clocks desynchronization
   - Equivalence principle
   - Clocks gravitational redshift
2. Theoretical background
   - Experimental data
   - Experimental data
3. Results
   - Analysis methods
   - Phase or frequency model
   - Number of stations
   - ISS orbit deterioration
4. Conclusion

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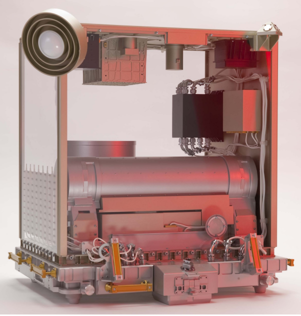
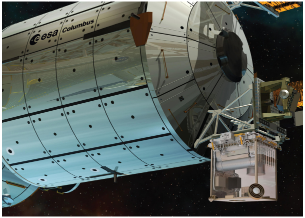
ACES-PHARAO mission

Objectives
Demonstrate ACES high performance and the ability to achieve high stability on space-ground time and frequency transfer.
Perform tests of fundamental physics at unprecedented accuracy





Launch date
Early 2020

Partners
CNES, ESA, industries and laboratories

Duration
18 months up to 3 years

Ph Laurent et al. 2015. The ACES/PHARAO space mission

Clocks desynchronization

Doppler effect

$$\nu_i = \nu_0 \frac{1 + \frac{v_i}{c}}{1 + \frac{v_O}{c}} \qquad (1)$$

$v < c$ and $v_0 = 0$

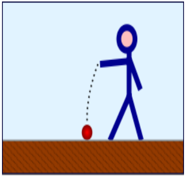
$$\nu_i = \nu_0 \left(1 + \frac{v_i}{c}\right) \qquad (2)$$

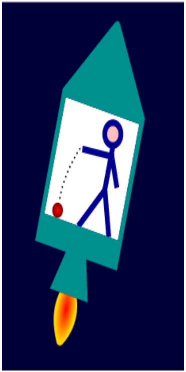
Frequency difference

$$\frac{\Delta\nu}{\nu} = \frac{\nu_2 - \nu_1}{\nu_0} = \frac{\Delta v}{c} \qquad (3)$$

Desynchronization

$$\frac{\Delta\nu}{\nu} \simeq \frac{a\Delta t}{c} = \frac{aL}{c^2} \qquad (4)$$





Two people drop an object, the first one is in a gravitational field ($\vec{g}$), the other is in a rocket with a constant acceleration ($\vec{a} = -\vec{g}$).

Equivalence principle

- we [...] assume the complete physical equivalence of a gravitational field and a corresponding acceleration of the reference system. -
Einstein, 1907

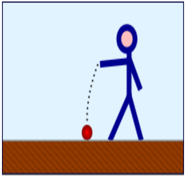
Equivalence principle decomposition :

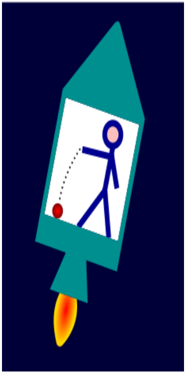
- Universality of Free Fall
- Lorentz invariance
- Local position invariance

Desynchronization : acceleration

$$\frac{\Delta\nu}{\nu} \simeq \frac{aL}{c^2}$$

Clocks gravitational redshift





Desynchronization : acceleration

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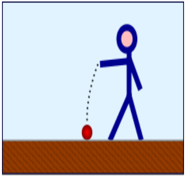

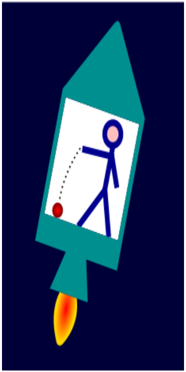

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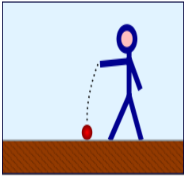

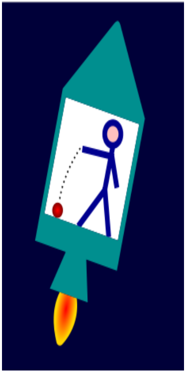

Desynchronization : acceleration

$$\frac{\Delta\nu}{\nu} \simeq \frac{aL}{c^2}$$

Clocks gravitational redshift

$$\frac{\Delta\nu}{\nu} \simeq \frac{gh}{c^2} \qquad \frac{\Delta\nu}{\nu}_{BigBen} = 5 \times 10^{-15} \quad (5)$$

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Test of the gravitational redshift : experimental data

Desynchronization model

$$\Delta\tau(t) = \Delta\tau_0 + \int_{t_0}^{t} \frac{V_{ground}^2 - V_{space}^2}{2c^2} \mathrm{d}t' + \int_{t_0}^{t} \frac{U_{ground} - U_{space}}{c^2} \mathrm{d}t' + \boxed{\alpha} \int_{t_0}^{t} \frac{U_{ground} - U_{space}}{c^2} \mathrm{d}t' \quad (6)$$

Gravity Probe A

$$\sigma_\alpha = 2 \times 10^{-4}$$

R. F. C. Vessot et al. 1979

Great Experiment

$$\sigma_\alpha = \text{LOW} \quad 10^{-5}$$

P. Delva et al. (2018)

Test of the gravitational redshift : model

Fitted model

$$
\begin{aligned}
Y(t) &= \Delta\tau(t) - \int_{t_0}^{t} \frac{V_{ground}^2 - V_{space}^2}{2c^2}\mathrm{d}t' - \int_{t_0}^{t} \frac{U_{ground} - U_{space}}{c^2}\mathrm{d}t' \\
&= \Delta\tau_0 + \boxed{\alpha} \int_{t_0}^{t} \frac{U_{ground} - U_{space}}{c^2}\mathrm{d}t' \\
&= \begin{pmatrix} 1 & \vdots & \vdots & \int_{t_0}^{t_1} \frac{\Delta U}{c^2}\mathrm{d}t' \\ \vdots & 1 & \vdots & \vdots \\ \vdots & \vdots & \vdots & \int_{t_0}^{t_n} \frac{\Delta U}{c^2}\mathrm{d}t' \end{pmatrix} \begin{pmatrix} \Delta\tau_0^{OPMT} \\ \Delta\tau_0^{PTBB} \\ \vdots \\ \boxed{\alpha} \end{pmatrix} \\
&= AX
\end{aligned} \tag{7}
$$

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Analysis methods

Method	Least squares Monte Carlo (LSMC)	Generalized least squares (GLS)
Model Solution Uncertainty	$Y = Ax$ $x = (A^T A)^{-1} A^T Y$ $\sigma_{LS} = \sigma_{noise}(A^T A)^{-1/2}$	$WY = WAx$ with W the inverse noise covariance matrix $x = (A^T WA)^{-1} A^T WY$ $\sigma_{GLS} = (A^T WA)^{-1/2}$
How to get σ_α ?	Standard deviation of a set of N_{MC} least squares simulation	σ_{GLS}
CPU (time)	N_{MC} linear dependance	Instantaneous
RAM (memory)	Data length : n	Storing and inverting W : n^2
Prerequisites	General noise characteristics	Good knowledge of W
Validity area		inversion of a $n \times n$ covariance matrix $\Rightarrow n < 10.000$
Uncertainty	Theoretically higher than GLS	Cramér-Rao bound : best estimator
Summary	No-Brainer is Simpler : Computer efficient implementation but leads to higher uncertainty	Brainer is Better : Best estimator but requires to know the covariance matrix

Results

$$\sigma_{LSMC} \simeq \sigma_{GLS} \tag{8}$$

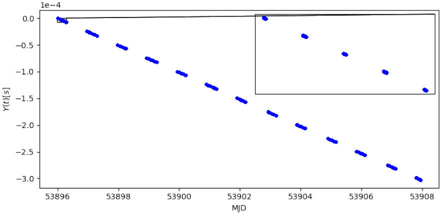
Phase or frequency model

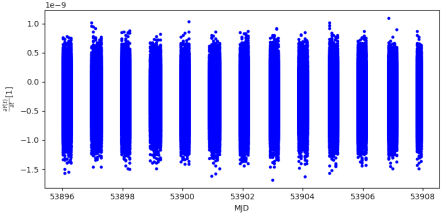
Phase model

$$Y(t) = \Delta\tau_0 + \boxed{\alpha} \int_{t_0}^{t} \frac{U_{ground} - U_{space}}{c^2} \mathrm{d}t' \qquad (9)$$

Frequency model

$$X(t) = \frac{dY}{dt} = \boxed{\alpha} \frac{U_{ground} - U_{space}}{c^2} \qquad (10)$$

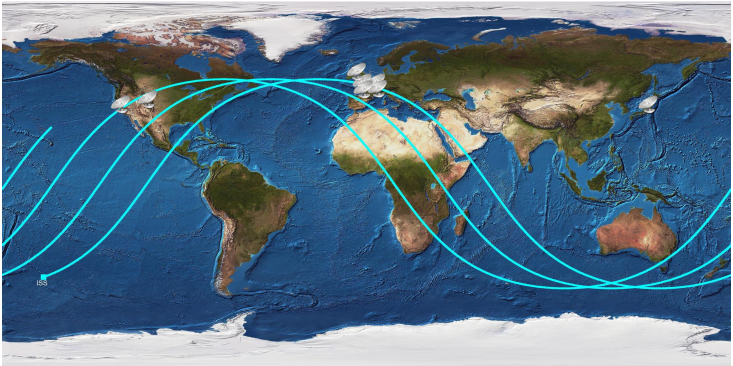




	Phase	Frequency
σ_α	3.1×10^{-6}	1.0×10^{-4}

Table: Uncertainty of α : phase vs frequency

Number of stations



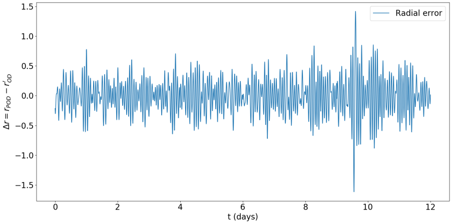
	1 station	2 stations	Full network
σ_α	4.2×10^{-6}	3.7×10^{-6}	3.1×10^{-6}

Table: Uncertainty of α : number of stations

ISS orbit deterioration

Scaling the ISS orbit error

- Precise orbit *POD*
- Less precise *OD*
- Orbit error $POD - OD$



Orbit file

$$OF = POD + k(POD - OD)$$

$k =$	0	1	10^3	10^4
Orbit error		1m	1km	10km
α	2×10^{-6}	2×10^{-6}	-1×10^{-6}	-3×10^{-5}
σ_α	4×10^{-6}	4×10^{-6}	4×10^{-6}	4×10^{-6}
Significant	False	False	False	True

Table: α : ISS orbit deterioration

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Analysis methods

We will use the most efficient method (LSMC) rather than the best possible estimator (GLS).

Phase or frequency analysis

Due to the mission specificity, we will use the phase model over the frequency.

Number of stations

Using one station or the full network leads to (almost) the same uncertainty.

ISS orbit deterioration

A 100m error on the ISS orbit will not affect the gravitational redshift test.

Final expected uncertainty

We will achieve :

$$\sigma_\alpha = 3 \times 10^{-6} \tag{11}$$

Test of the gravitational redshift : modified theory

Hypothesis A body's mass is modified by the energy needed to keep its structure and composition.

Modified mass

$$m = m_0 - \delta m_I \frac{V^2}{2c^2} - \delta m_P \frac{U}{c^2} \qquad (12)$$

with δm_X modificiations to the normal mass m_0.

The energy becomes :

Modified energy

$$E = m_0 c^2 + \frac{1}{2} m_0 \left(1 + \frac{\delta m_I}{m_0}\right) V^2 - m_0 \left(1 + \frac{\delta m_P}{m_0}\right) U \qquad (13)$$

Test of the gravitational redshift : modified metric

Two levels system, $\delta m_I = 0$

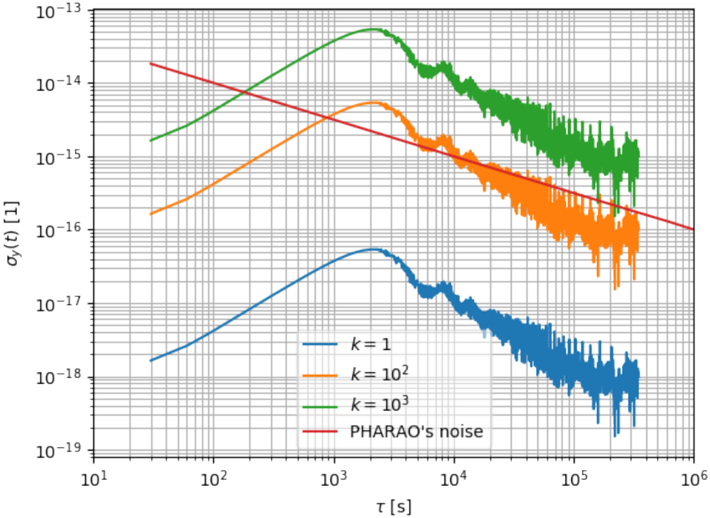
Modified metric

$$\frac{\nu}{\nu_0} = \frac{d\tau}{dt} = 1 - \frac{V^2}{2c^2} - \frac{U}{c^2} - \alpha_P \frac{U}{c^2} \tag{14}$$

Schwartzchild metric and Doppler effect

$$\frac{d\tau}{dt} = 1 - \frac{V^2}{2c^2} - \frac{U}{c^2} \tag{15}$$

ISS orbit deterioration



Condition

ORBIT ERROR $< 1 km$