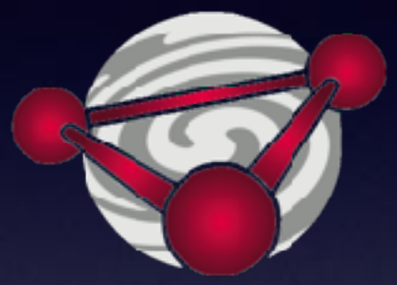


Gravitational Waves

Laser Noise Reduction for LISA

Jean-Baptiste Bayle

GPhys Day 2018

Let Me Walk You Through...

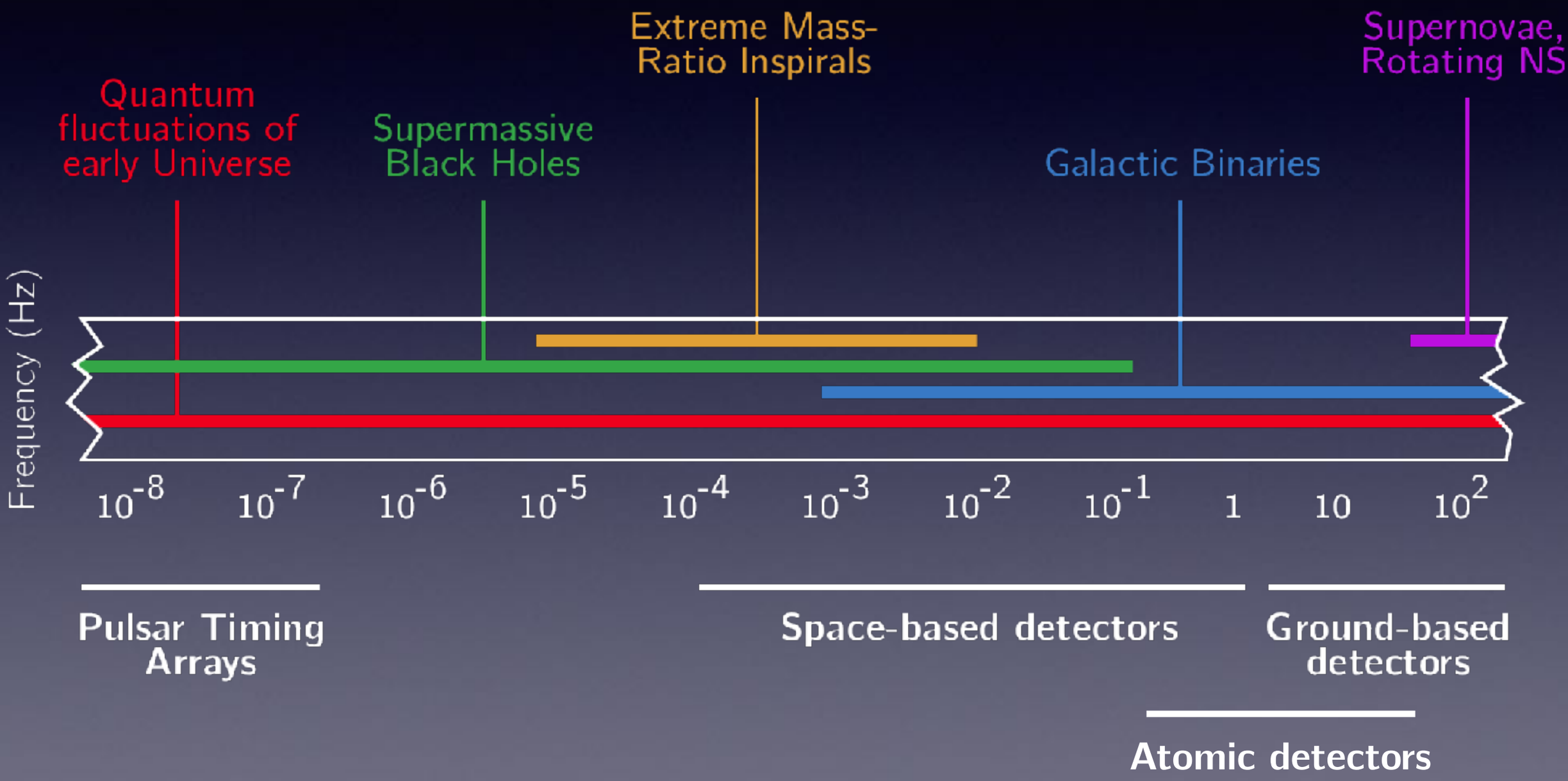
Space-Based Detection of Gravitational Waves

Laser Noise and Time-Delay Interferometry

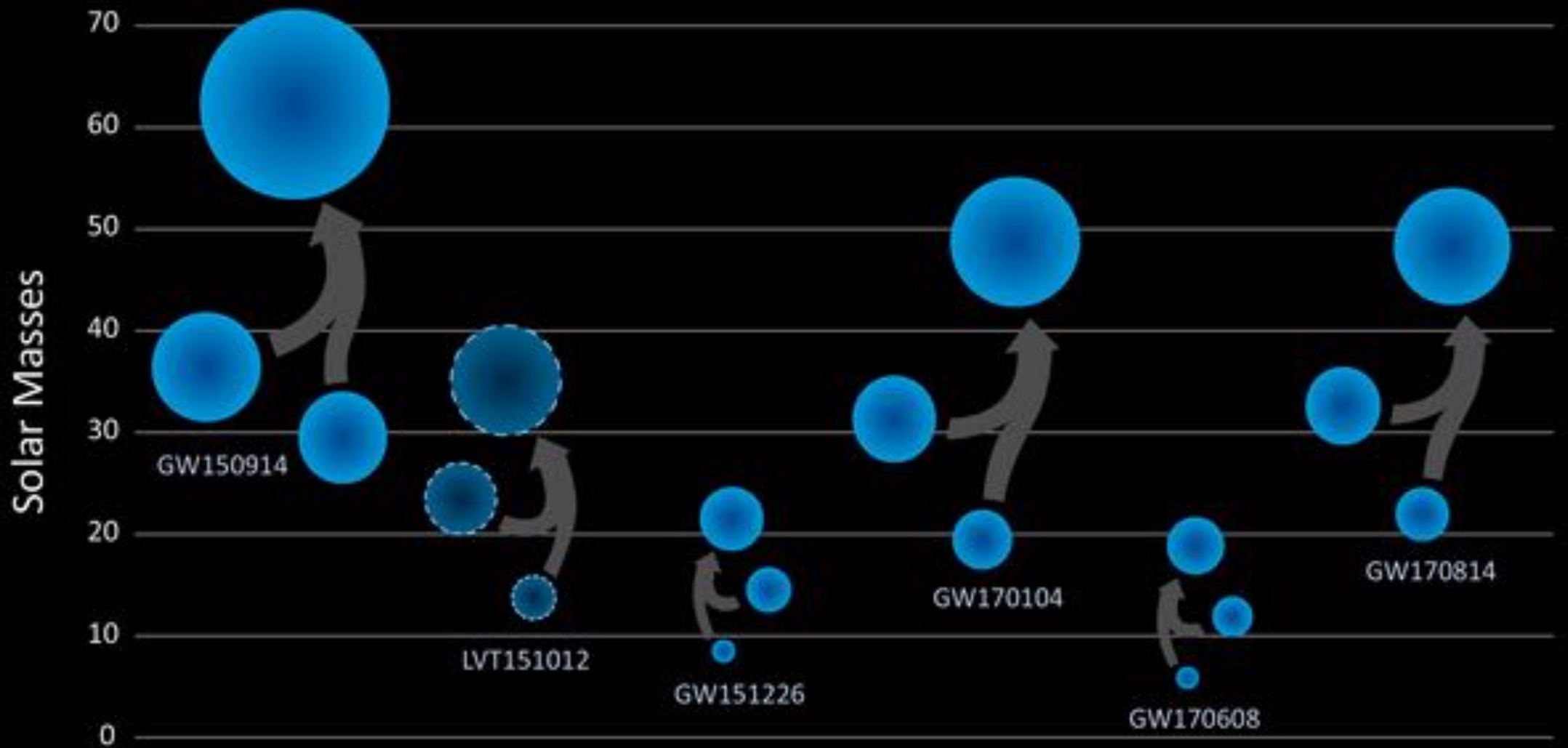
Impact on LISA Science and Instrument Design

SPACE-BASED DETECTION OF GRAVITATIONAL WAVES

Gravitational Spectrum



First LIGO-Virgo detections



LIGO-Virgo Collaboration

Gravitation from space

- Exploring the lower-frequency part requires to **go to space**: this is ESA's **Laser Interferometer Space Antenna** (LISA) project

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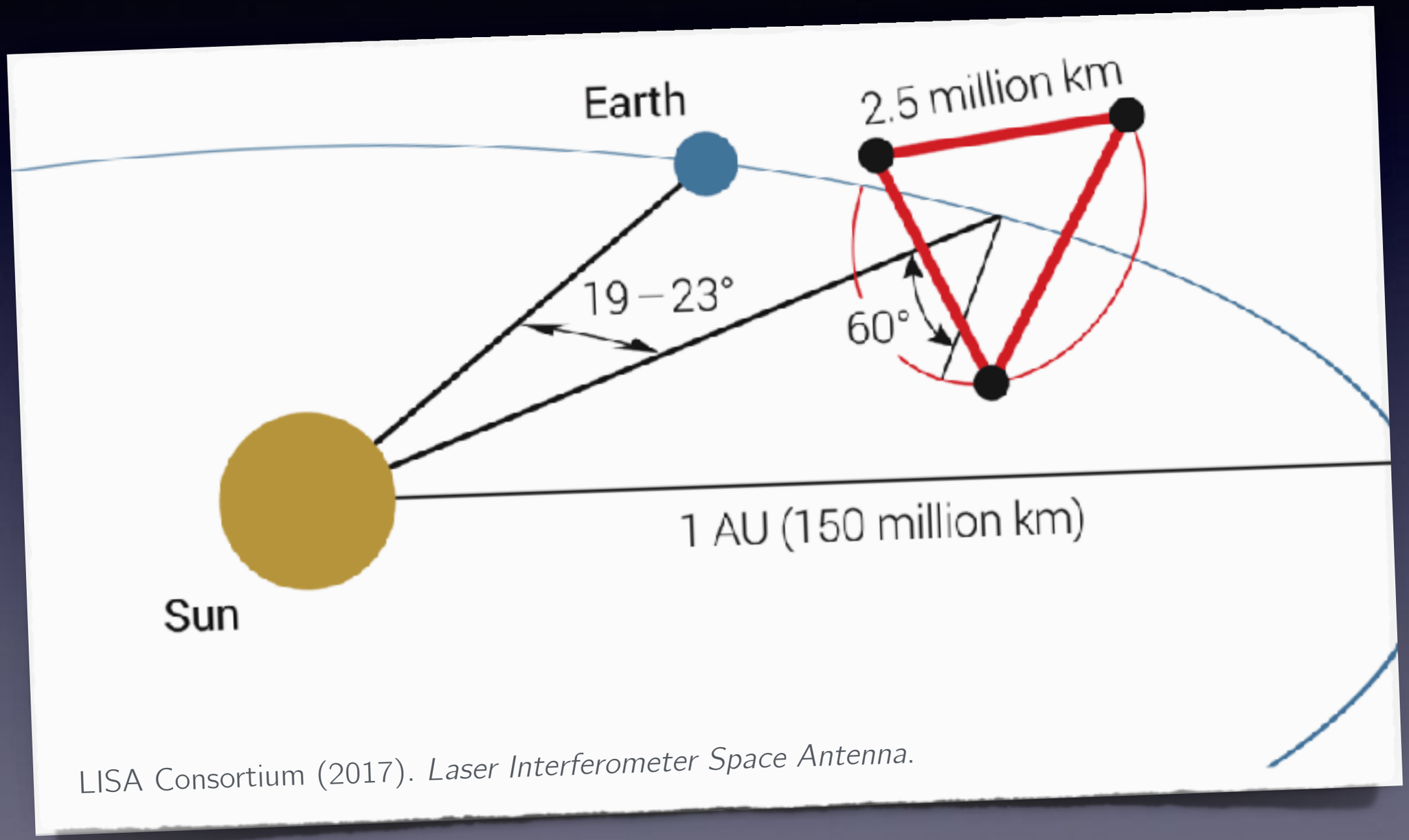
- Trailing the Earth on its orbit around the Sun and rotating through the three years of the nominal mission

Gravitation from space



LISA Consortium (2017). *Laser Interferometer Space Antenna*.

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Gravitation from space

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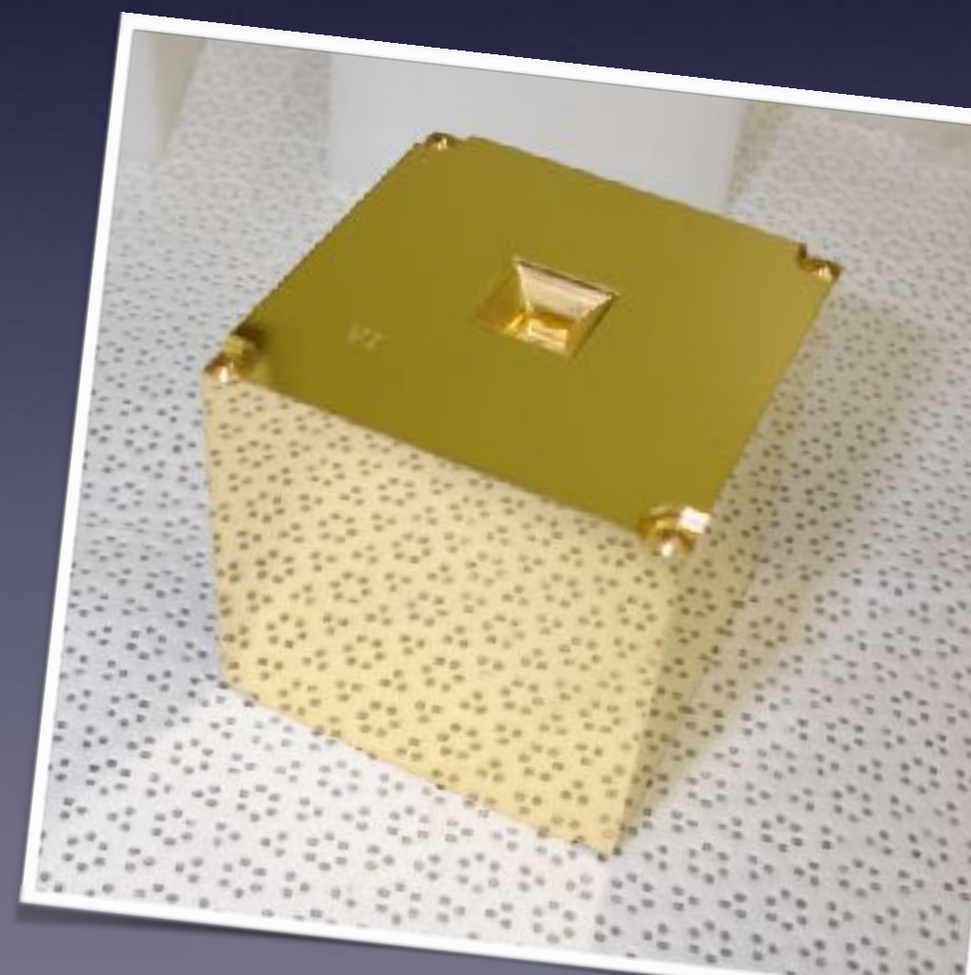
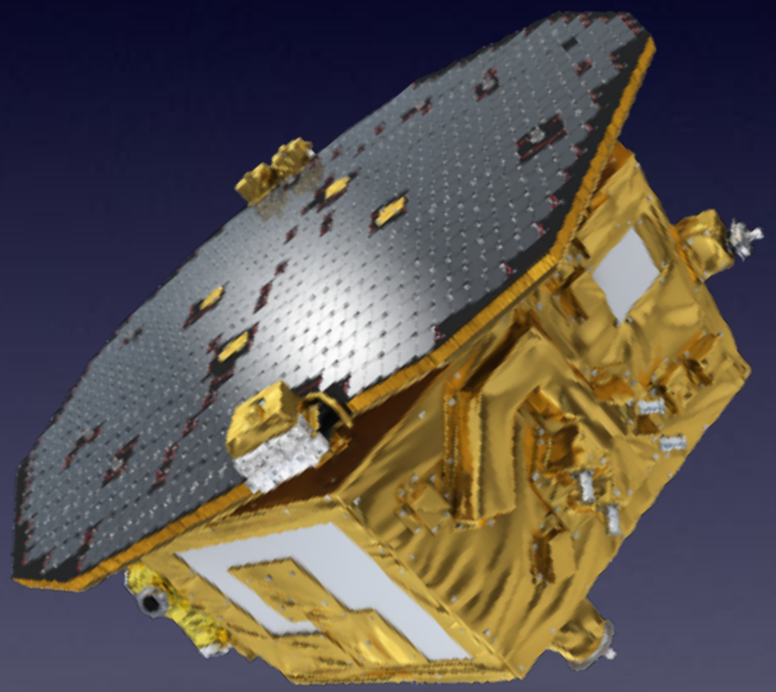
- Spacecraft submitted to solar winds, radiation pressure, thrusters noise, etc...
- Gold-platinum cubic **test-masses** protected inside spacecraft and therefore kept **drag-free**, so they can fly on geodesics




Credits: ESA

LISAPathFinder: a proof of concept

- Proof-of-concept ESA mission launched in December 2015



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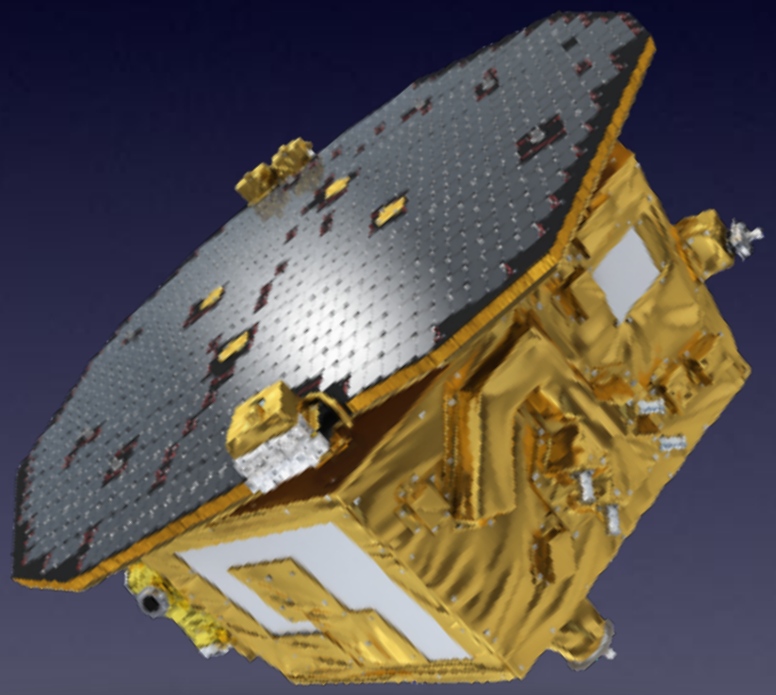

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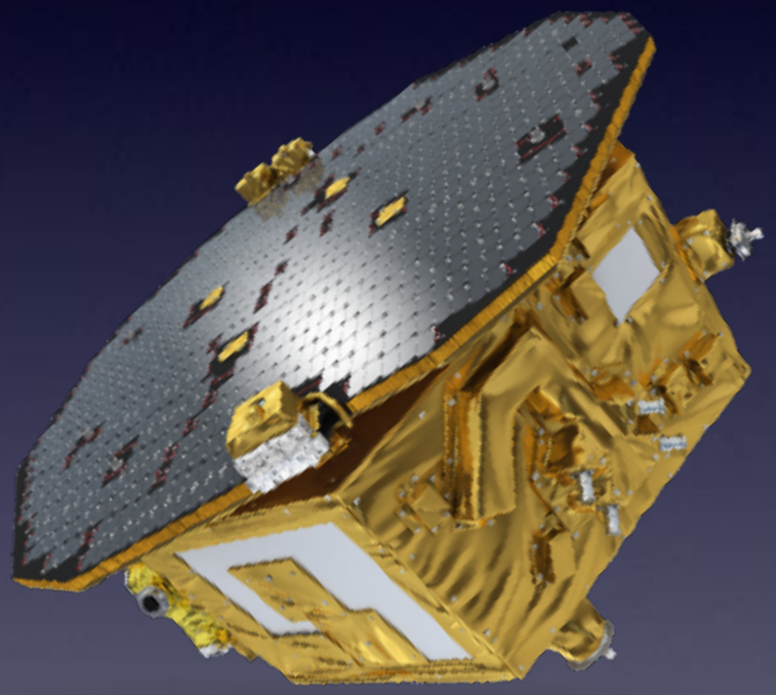
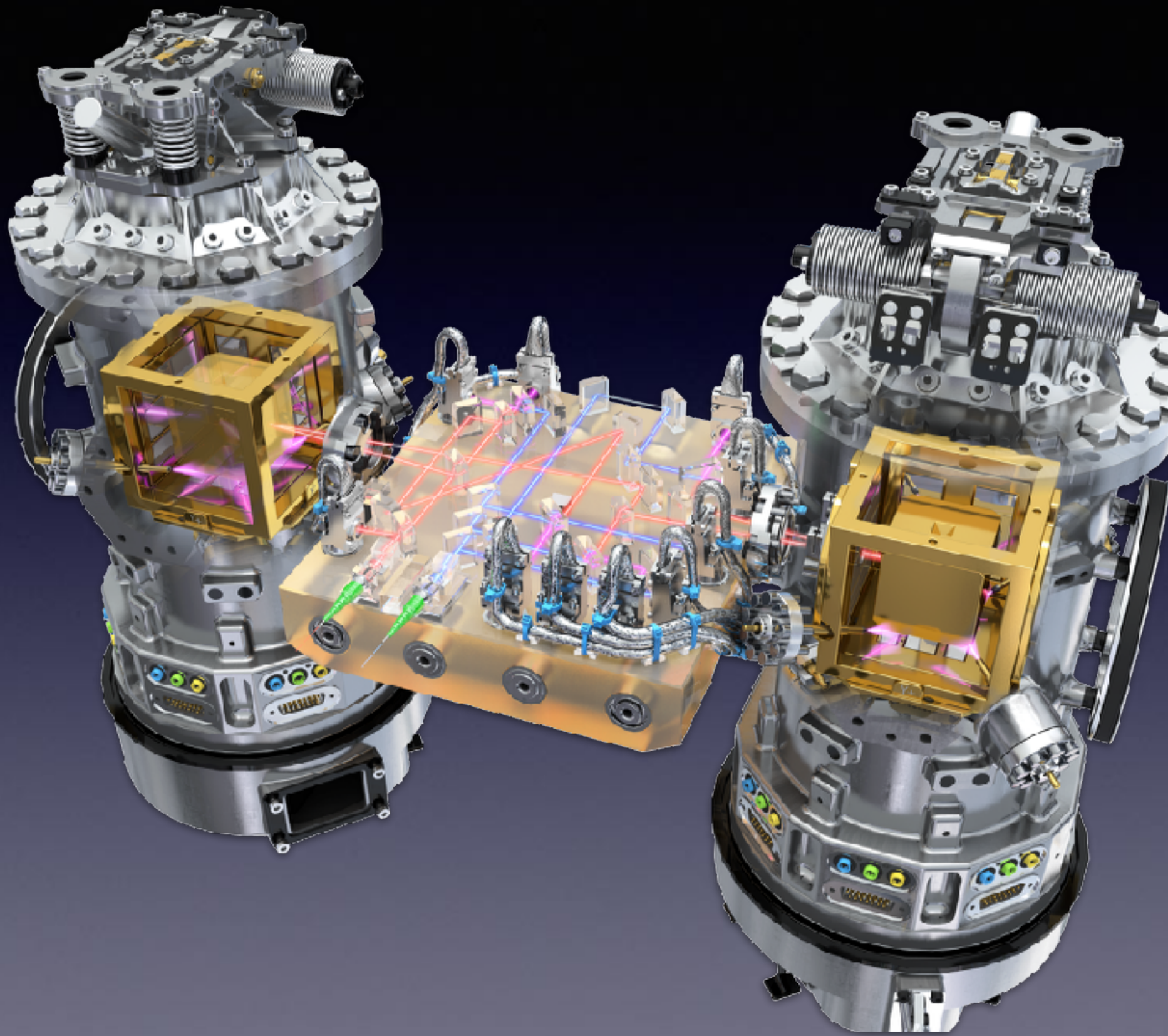


- Proof-of-concept ESA mission launched in December 2015
- An LISA arm is shrunk down to **38 cm** between two test masses that are kept drag-free
- Optical interferometry for precise measurement of the **residual acceleration** of one mass relative to the other



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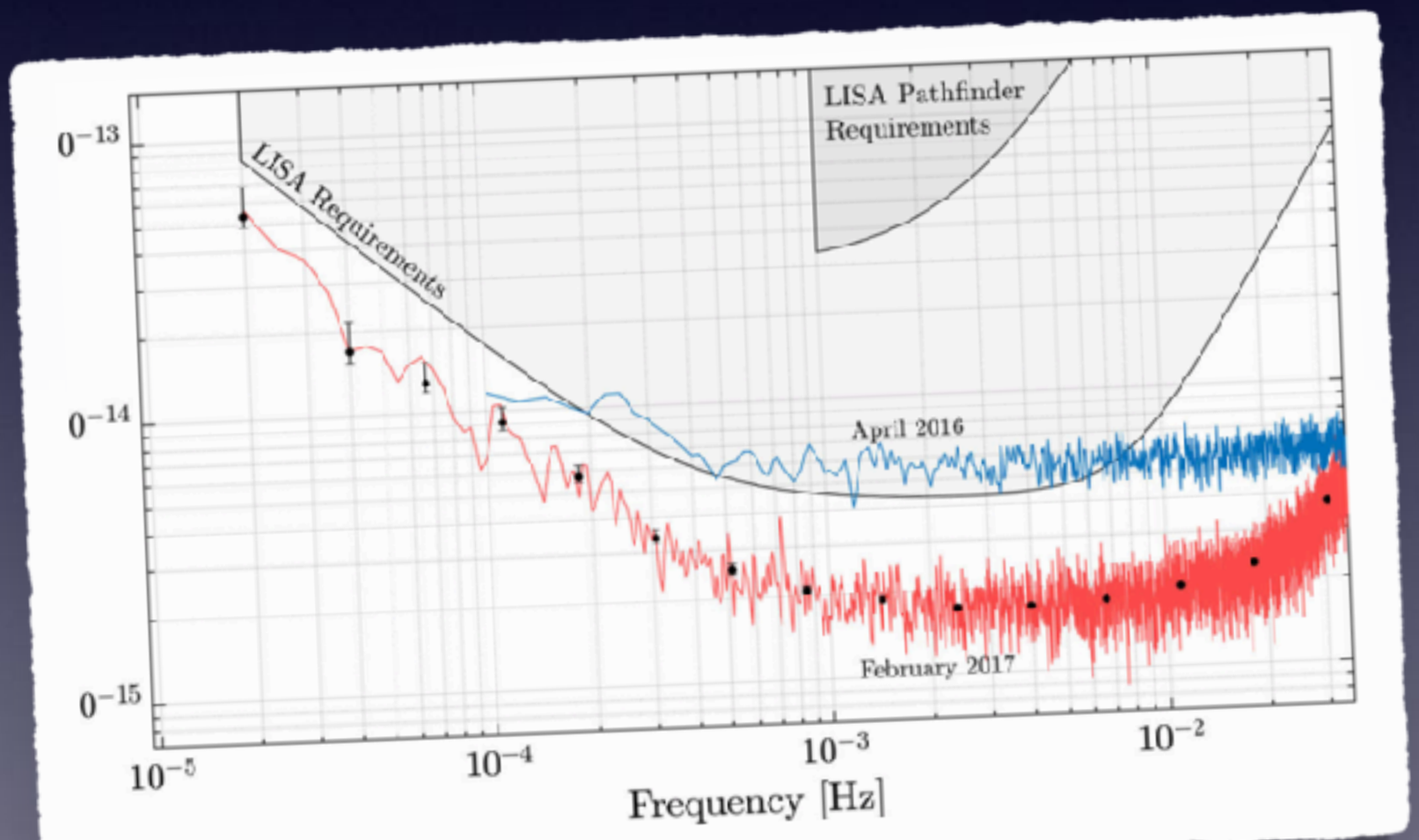
LISAPathFinder: a proof of concept

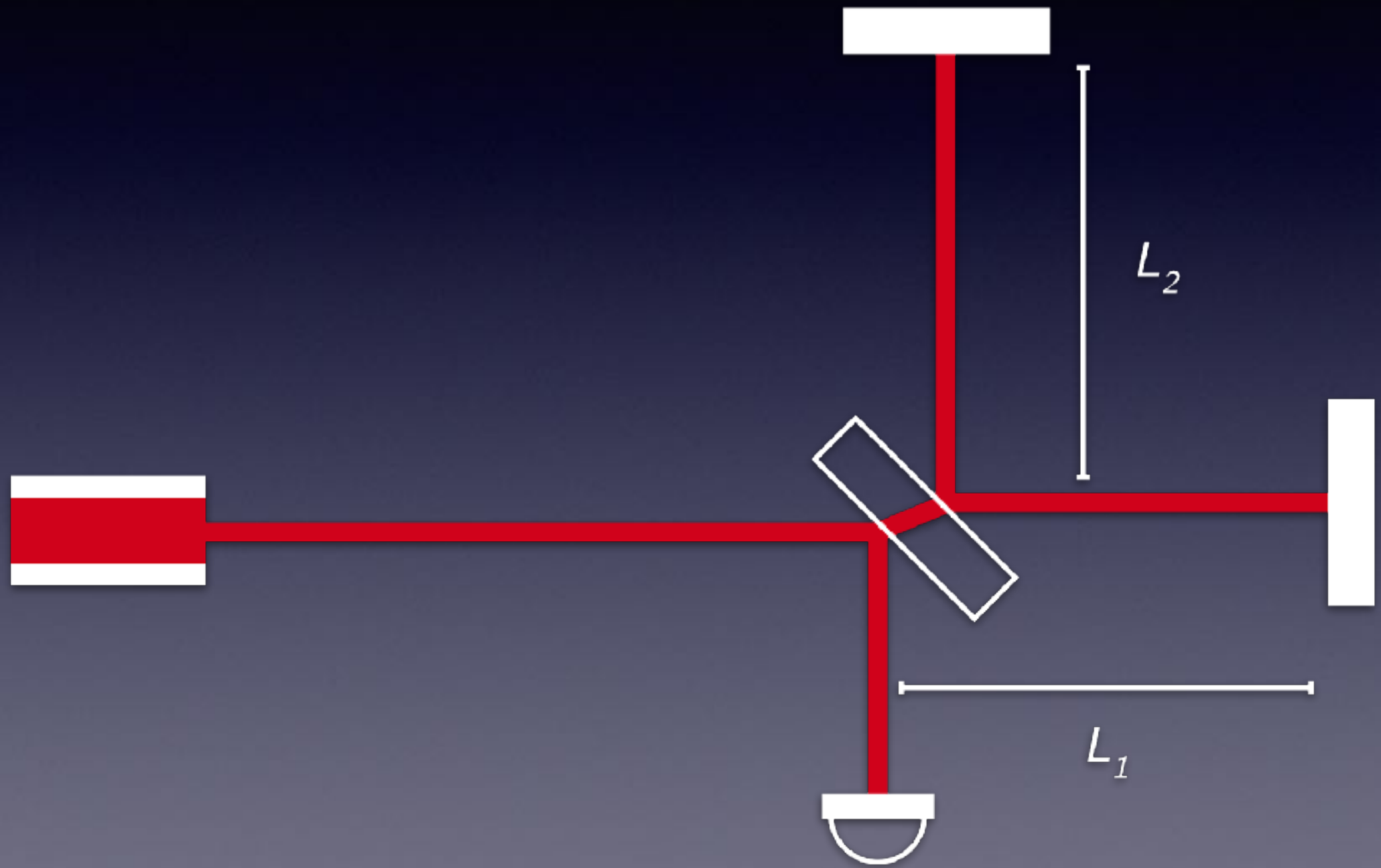
- **Results** announced last February exceeded requirements in the LISAPathFinder frequency band



Armano, M., et al. (2018). Beyond the Required LISA Free-Fall Performance: New LISA Pathfinder Results down to 20 μHz. *Physical Review Letters*, *120*(6), 061101

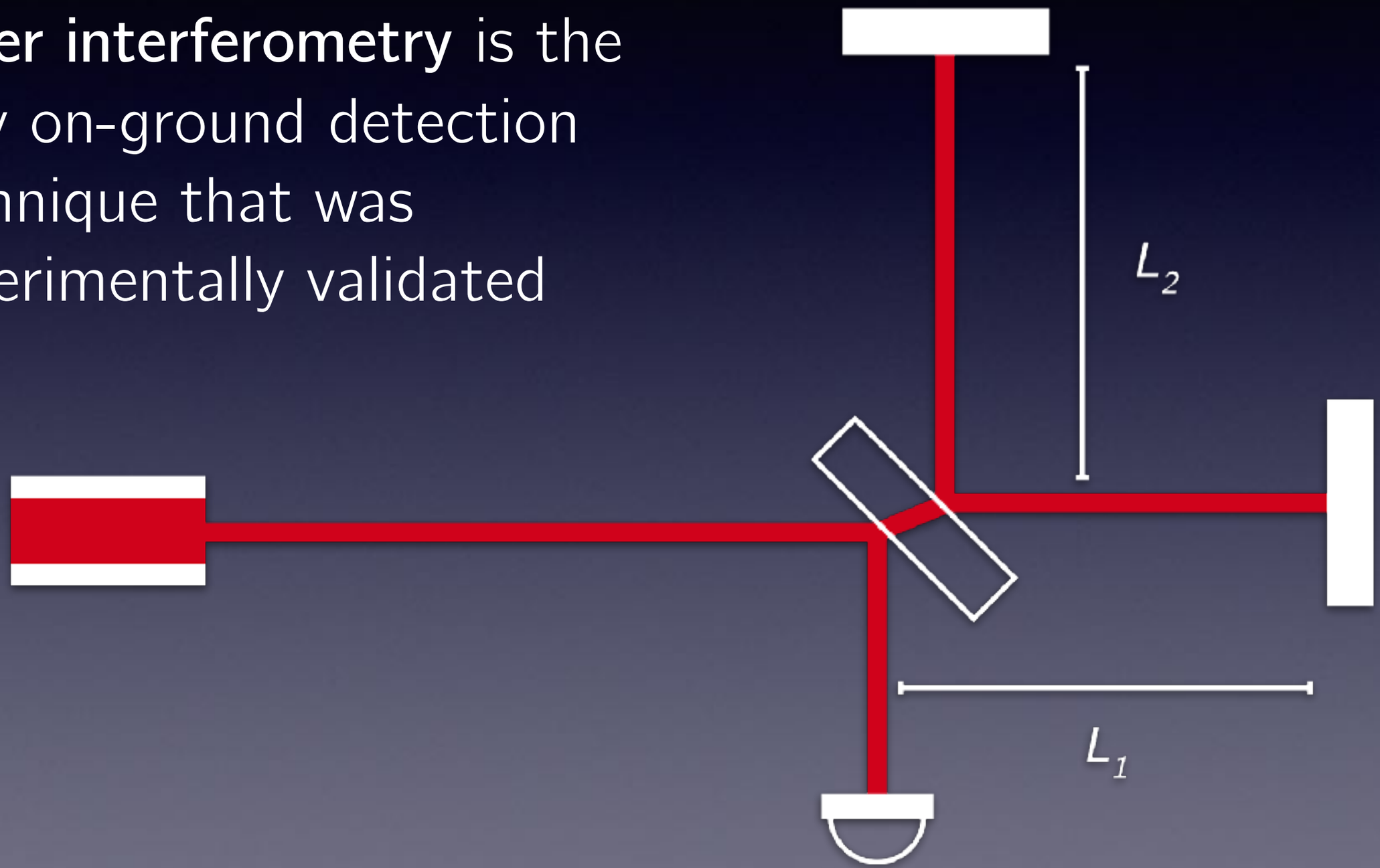
LASER NOISE AND TIME-DELAY INTERFEROMETRY

Interferometric Detection



Interferometric Detection

- **Laser interferometry** is the only on-ground detection technique that was experimentally validated



Interferometric Detection

$$P(t) \propto 1 + \cos\left(\Delta\phi\right)$$

$$\Delta\phi \frac{1}{\omega_{\mathrm{GW}}} \sin\left(\frac{\omega_{\mathrm{GW}} L}{c}\right) \cos(\omega_{\mathrm{GW}} t)$$

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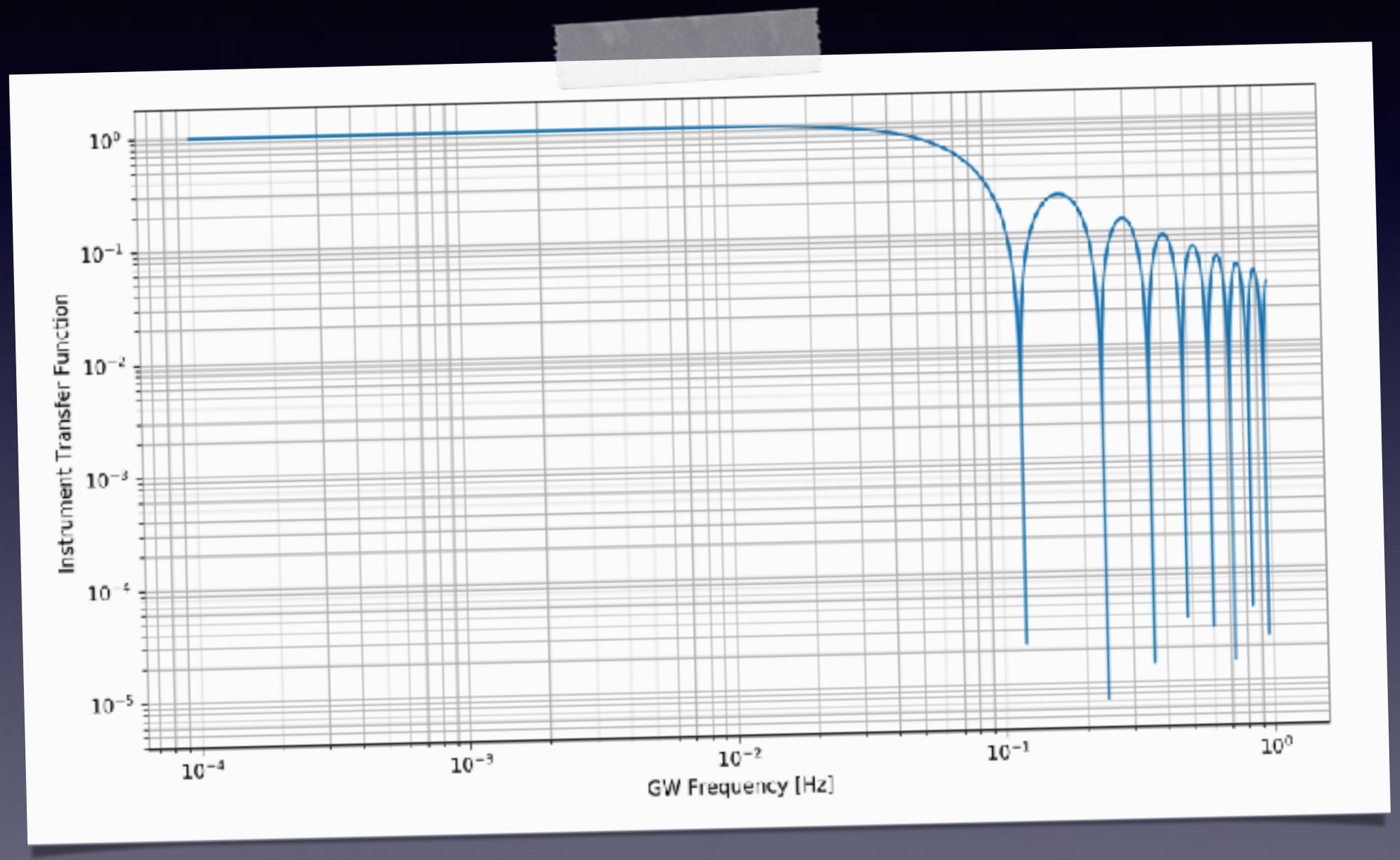

Michelson Transfer Function for Monochromatic Waves

Space-Based Detection

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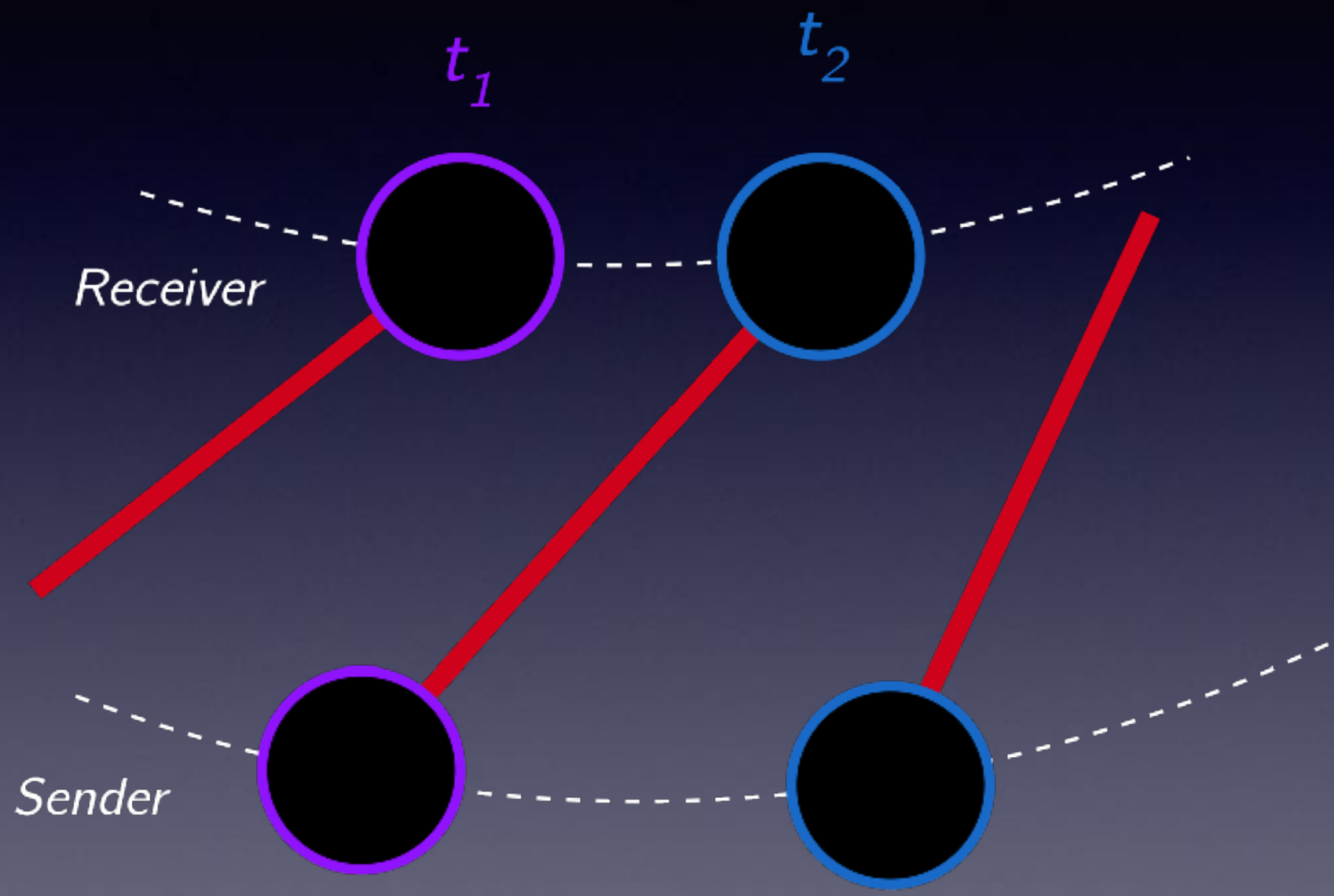
- Point-ahead mechanism and moving telescopes and optical benches to be sure to send beam correctly
- Additional **shift in phase** due to relativistic effects and **Doppler effect** that can mock passing waves

Space-Based Detection



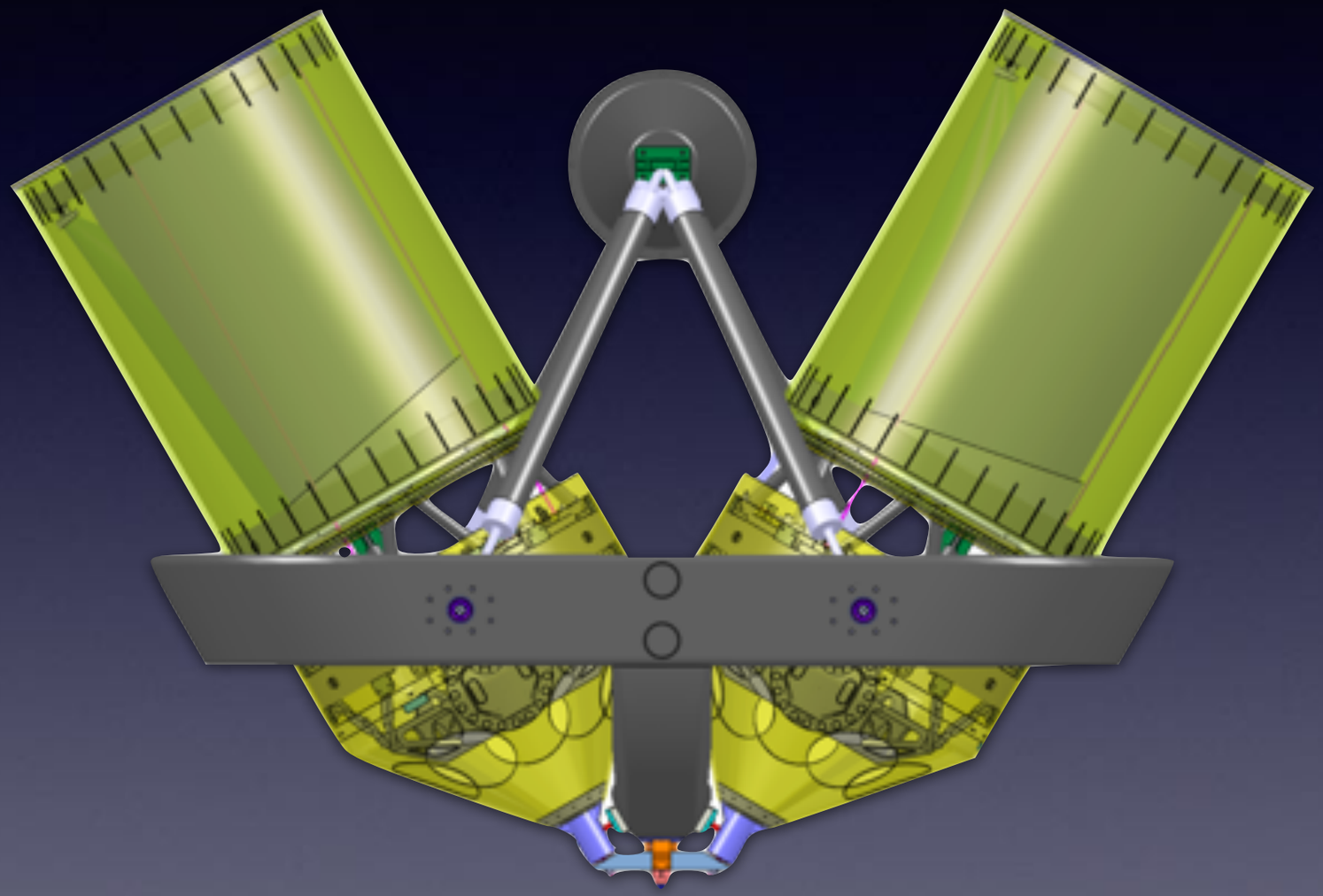
Point-Ahead Mechanism

Space-Based Detection

Moving Telescopes, Optical Benches and GRS

Credits: ESA

Space-Based Detection



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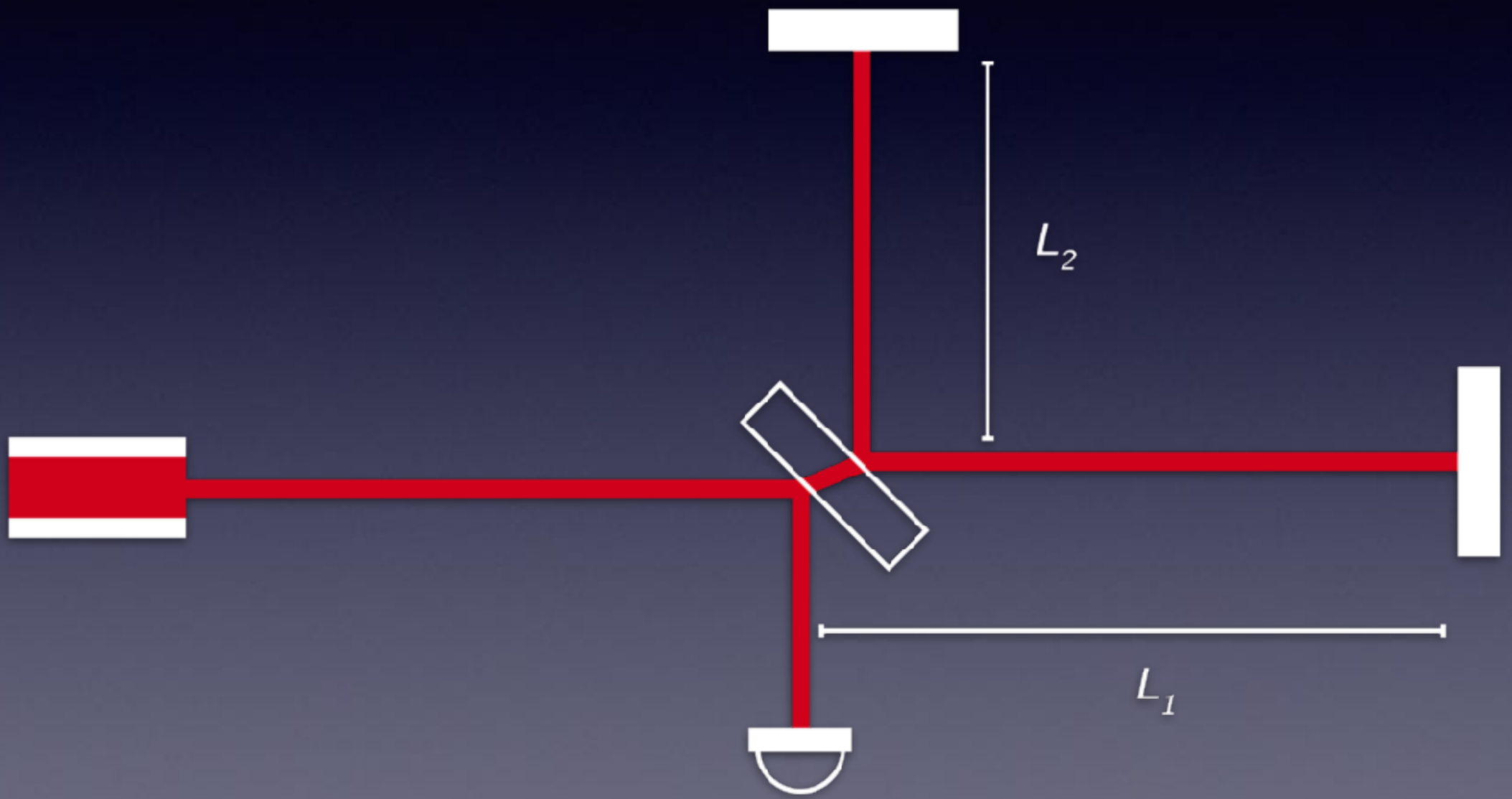
Laser Noise with Unequal Arms



Laser Noise with Unequal Arms

$$y_1(t) = H_1(t) + \omega\left(t - \frac{2L_1}{c}\right) + p\left(t - \frac{2L_1}{c}\right)$$

$$y_2(t) = H_2(t) + \omega\left(t - \frac{2L_2}{c}\right) + p\left(t - \frac{2L_2}{c}\right)$$

$$y(t) = H_1(t) - H_2(t) + p\left(t - \frac{2L_1}{c}\right) - p\left(t - \frac{2L_2}{c}\right)$$

Laser Noise with Unequal Arms

- **Laser frequency noise** does not cancel to levels that allow detection of gravitational waves

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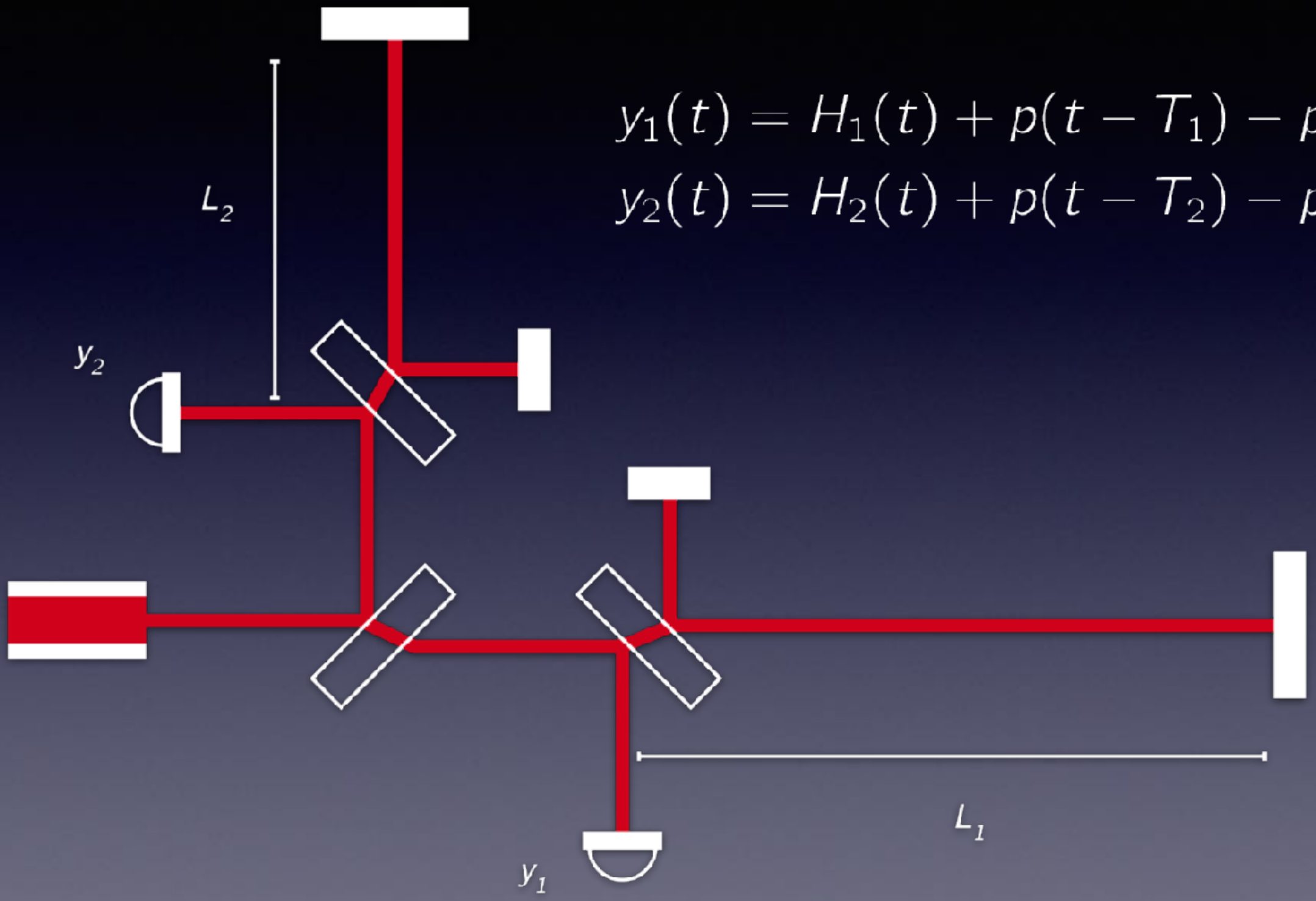
- Phase must be measured at **µrad precision** so we need a post-precessing algorithm to **reduce this laser frequency noise by 8 orders of magnitude in amplitude**
- This is **Time-Delay Interferometry (TDI)**, described in [Armstrong, Estabrook and Tinto. 1999]

$$y_1(t) = H_1(t) + p(t - T_1) - p(t)$$
$$y_2(t) = H_2(t) + p(t - T_2) - p(t)$$



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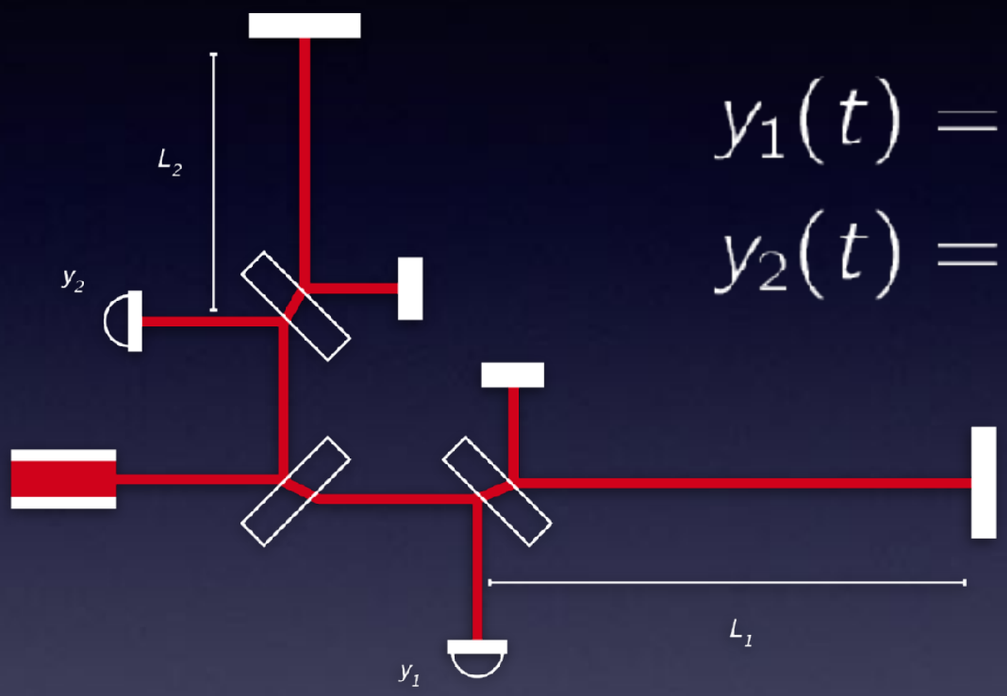

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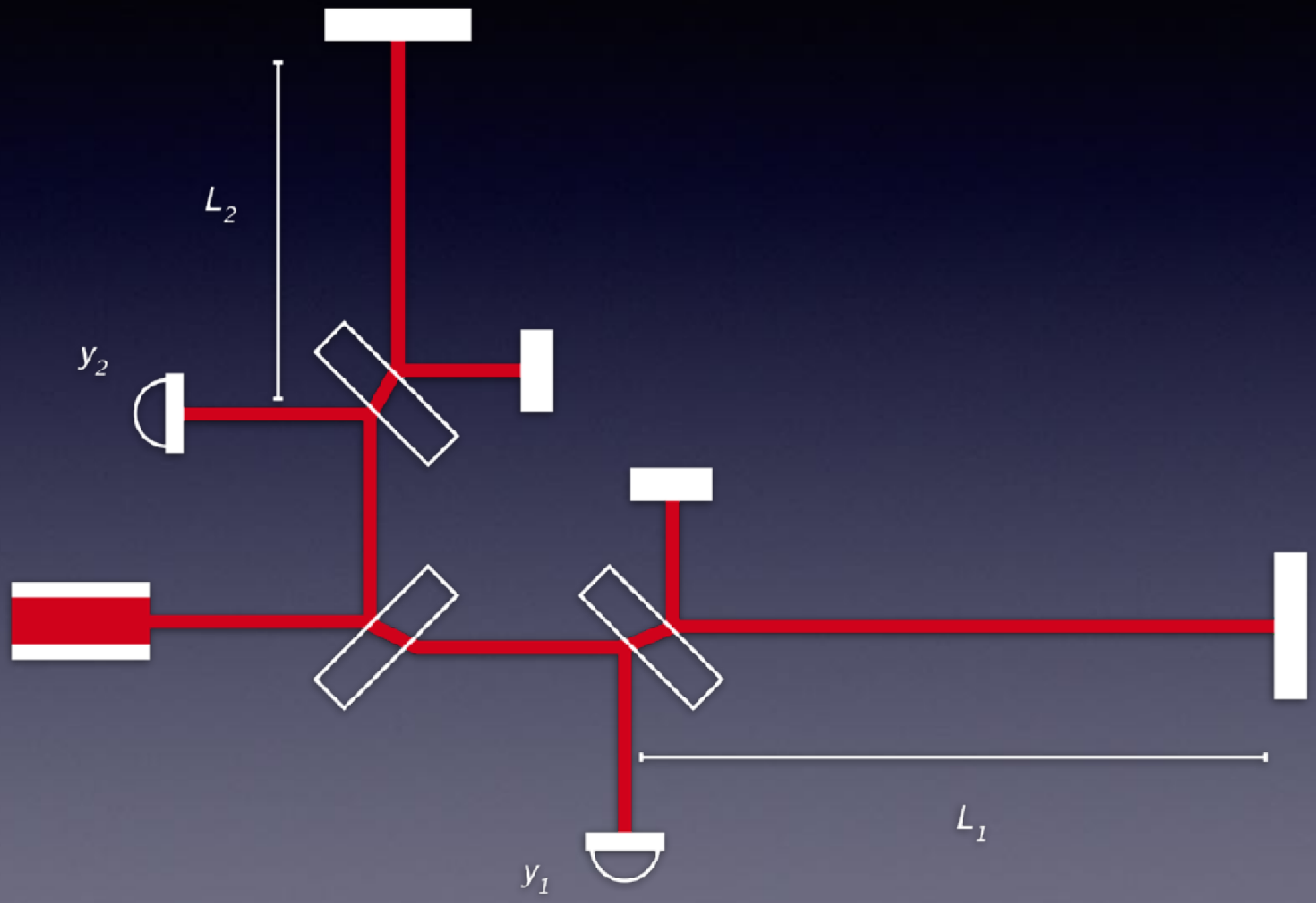
$$y_1(t - T_2) - y_2(t - T_1) = \cdots + p(t - T_1) - p(t - T_2)$$

$$y(t) = [y_1(t) - y_2(t)] - [y_1(t - T_2) - y_2(t - T_1)]$$

$$y(t) = [y_2(t) + y_1(t - T_2)] - [y_1(t) + y_2(t - T_1)]$$



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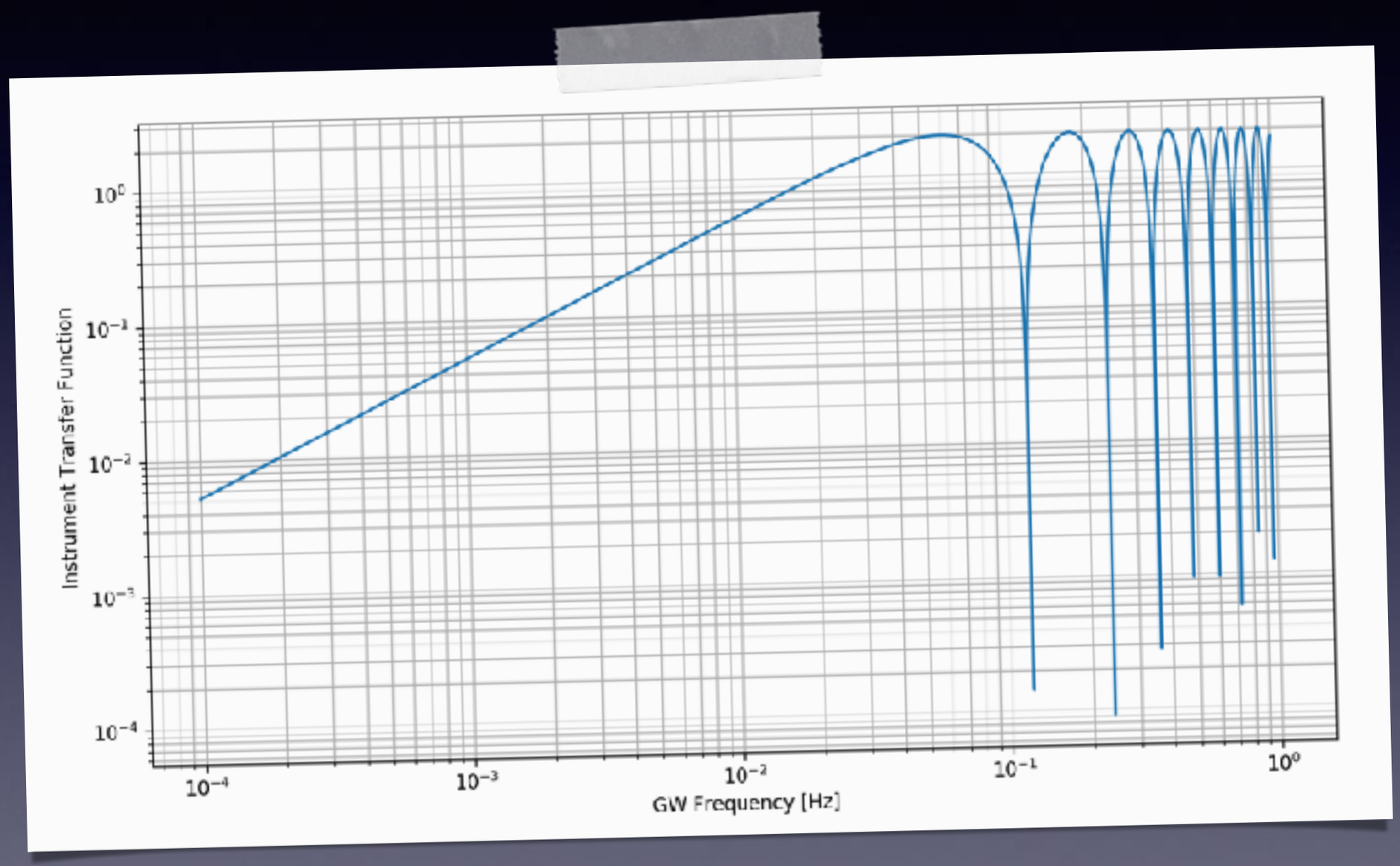

TDI-like Transfer Function for Monochromatic Waves

LISA Time-Delay Interferometry

$$X_{1.5} = s_1 + \mathcal{D}_3 s_2' + \mathcal{D}_3\mathcal{D}_{3'} s_1' + \mathcal{D}_3\mathcal{D}_{3'}\mathcal{D}_{2'} s_3$$
$$- s_1' - \mathcal{D}_{2'} s_3 - \mathcal{D}_{2'}\mathcal{D}_2 s_1 - \mathcal{D}_{2'}\mathcal{D}_2\mathcal{D}_3 s_2'$$

LISA Time-Delay Interferometry

- For LISA we reconstruct 3 offline Michelson interferometers, among which two independent

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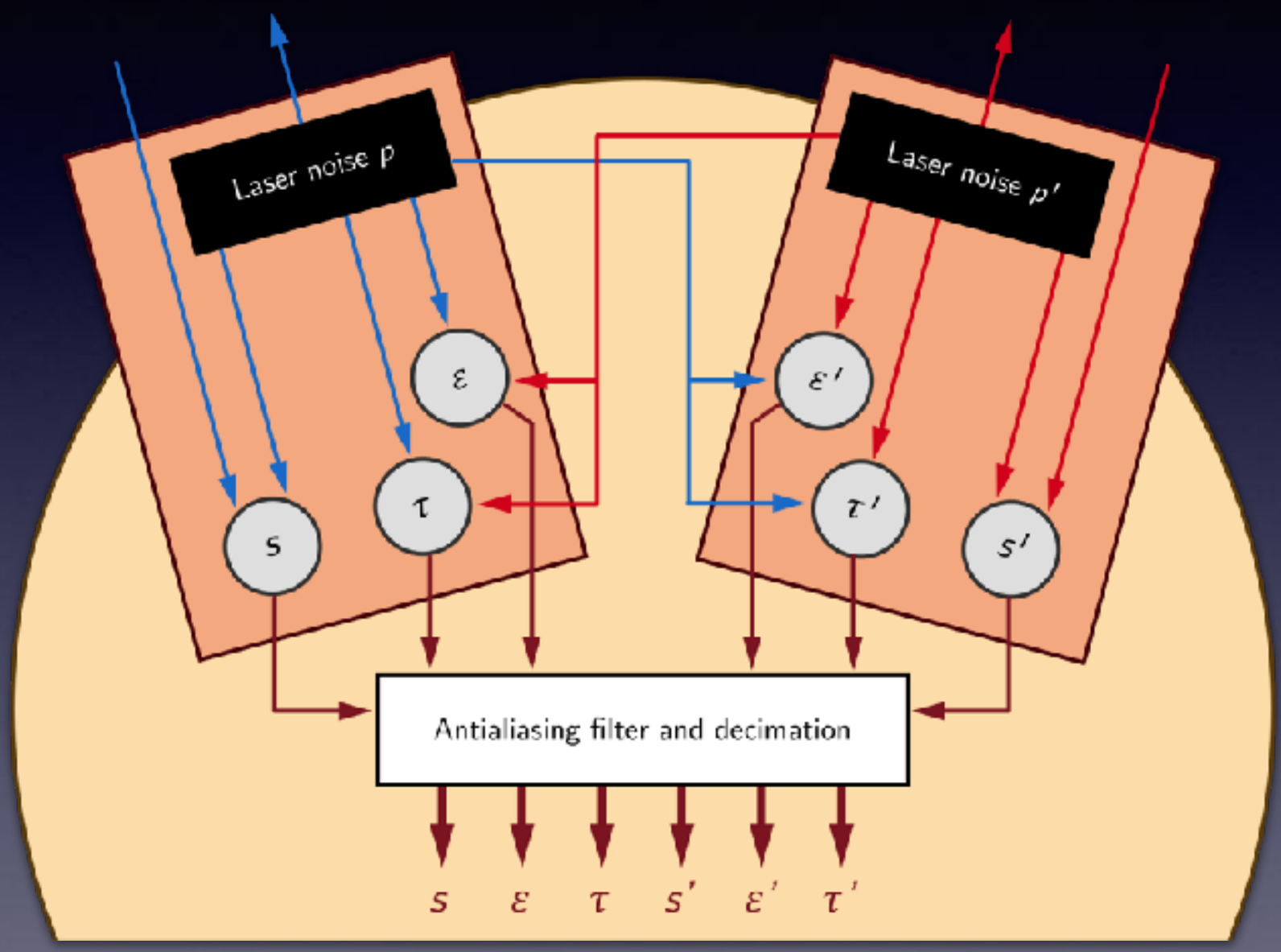
- **3 interferometric measurements** per optical bench: s for science, τ for reference, ε for test-mass
- Used to reduce other correlated noises

LISA Time-Delay Interferometry



Optical Setup for a LISA Spacecraft

LISA Time-Delay Interferometry

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- **Effect of antialiasing filters**?

$$
\begin{aligned}
X_2 = {} & (\mathcal{D}_3\mathcal{F}\mathcal{D}_{3'} - \mathcal{D}_3\mathcal{D}_{3'}\mathcal{F})p_1 + (\mathcal{D}_3\mathcal{D}_{3'}\mathcal{D}_{2'}\mathcal{D}_2\mathcal{F} - \mathcal{D}_{2'}\mathcal{D}_2\mathcal{D}_3\mathcal{F}\mathcal{D}_{3'})p_1 \\
& + (\mathcal{D}_{2'}\mathcal{D}_2\mathcal{F} - \mathcal{D}_{2'}\mathcal{F}\mathcal{D}_2)p_1' + (\mathcal{D}_3\mathcal{D}_{3'}\mathcal{D}_{2'}\mathcal{F}\mathcal{D}_2 - \mathcal{D}_3\mathcal{D}_{3'}\mathcal{D}_{2'}\mathcal{D}_2\mathcal{F})p_1' \\
& + (\mathcal{F}\mathcal{D}_3 - \mathcal{D}_3\mathcal{F})p_2' + (\mathcal{D}_{2'}\mathcal{D}_2\mathcal{D}_3\mathcal{F} - \mathcal{D}_{2'}\mathcal{D}_2\mathcal{F}\mathcal{D}_3)p_2' \\
& + (\mathcal{D}_{2'}\mathcal{F} - \mathcal{F}\mathcal{D}_{2'})p_3 + (\mathcal{D}_3\mathcal{D}_{3'}\mathcal{F}\mathcal{D}_{2'} - \mathcal{D}_3\mathcal{D}_{3'}\mathcal{D}_{2'}\mathcal{F})p_3 \\
& + (\mathcal{D}_{2'}\mathcal{D}_2\mathcal{D}_3\mathcal{D}_{3'}\mathcal{F} - \mathcal{D}_3\mathcal{D}_{3'}\mathcal{D}_{2'}\mathcal{D}_2\mathcal{F})p_1 \\
& + (\mathcal{D}_{2'}\mathcal{D}_2\mathcal{D}_3\mathcal{D}_{3'}\mathcal{D}_3\mathcal{D}_{3'}\mathcal{F} - \mathcal{D}_{2'}\mathcal{D}_2\mathcal{D}_3\mathcal{D}_{3'}\mathcal{D}_3\mathcal{F}\mathcal{D}_{3'})p_1 \\
& + (\mathcal{D}_3\mathcal{D}_{3'}\mathcal{D}_{2'}\mathcal{D}_2\mathcal{D}_{2'}\mathcal{D}_2\mathcal{D}_3\mathcal{F}\mathcal{D}_{3'} - \mathcal{D}_{2'}\mathcal{D}_2\mathcal{D}_3\mathcal{D}_{3'}\mathcal{D}_3\mathcal{D}_{3'}\mathcal{D}_{2'}\mathcal{D}_2\mathcal{F})p_1 \\
& + (\mathcal{D}_3\mathcal{D}_{3'}\mathcal{D}_{2'}\mathcal{D}_2\mathcal{D}_{2'}\mathcal{F}\mathcal{D}_2 - \mathcal{D}_3\mathcal{D}_{3'}\mathcal{D}_{2'}\mathcal{D}_2\mathcal{D}_{2'}\mathcal{D}_2\mathcal{F})p_1' \\
& + (\mathcal{D}_{2'}\mathcal{D}_2\mathcal{D}_3\mathcal{D}_{3'}\mathcal{D}_3\mathcal{D}_{3'}\mathcal{D}_{2'}\mathcal{D}_2\mathcal{F} - \mathcal{D}_{2'}\mathcal{D}_2\mathcal{D}_3\mathcal{D}_{3'}\mathcal{D}_3\mathcal{D}_{3'}\mathcal{D}_{2'}\mathcal{F}\mathcal{D}_2)p_1' \\
& + (\mathcal{D}_{2'}\mathcal{D}_2\mathcal{D}_3\mathcal{D}_{3'}\mathcal{D}_3\mathcal{F} - \mathcal{D}_{2'}\mathcal{D}_2\mathcal{D}_3\mathcal{D}_{3'}\mathcal{F}\mathcal{D}_3)p_2' \\
& + (\mathcal{D}_3\mathcal{D}_{3'}\mathcal{D}_{2'}\mathcal{D}_2\mathcal{D}_{2'}\mathcal{D}_2\mathcal{F}\mathcal{D}_3 - \mathcal{D}_3\mathcal{D}_{3'}\mathcal{D}_{2'}\mathcal{D}_2\mathcal{D}_{2'}\mathcal{D}_2\mathcal{D}_3\mathcal{F})p_2' \\
& + (\mathcal{D}_3\mathcal{D}_{3'}\mathcal{D}_{2'}\mathcal{D}_2\mathcal{F}\mathcal{D}_{2'} - \mathcal{D}_3\mathcal{D}_{3'}\mathcal{D}_{2'}\mathcal{D}_2\mathcal{D}_{2'}\mathcal{F} - \mathcal{D}_3\mathcal{D}_{3'}\mathcal{D}_{2'}\mathcal{D}_2\mathcal{F}\mathcal{D}_{2'})p_3 \\
& + (\mathcal{D}_{2'}\mathcal{D}_2\mathcal{D}_3\mathcal{D}_{3'}\mathcal{D}_3\mathcal{D}_{3'}\mathcal{D}_{2'}\mathcal{F} - \mathcal{D}_{-2}\mathcal{D}_2\mathcal{D}_3\mathcal{D}_{3'}\mathcal{D}_3\mathcal{D}_{3'}\mathcal{F}\mathcal{D}_{2'})p_3
\end{aligned}
$$

Simulation with LISA Node

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- Flexible **high-level simulation tool** based on graphs composed of atomic computation nodes

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Modeling of Residuals

$$S_{X_{1.5}}(\omega) = 16\omega^2 S_p(L_3\dot{L_2} - L_2\dot{L_3})^2$$

Modeling of Residuals

- Derived **analytic model for the laser noise residual power spectrum after TDI**, taking into account varying armlengths and antialiasing filters

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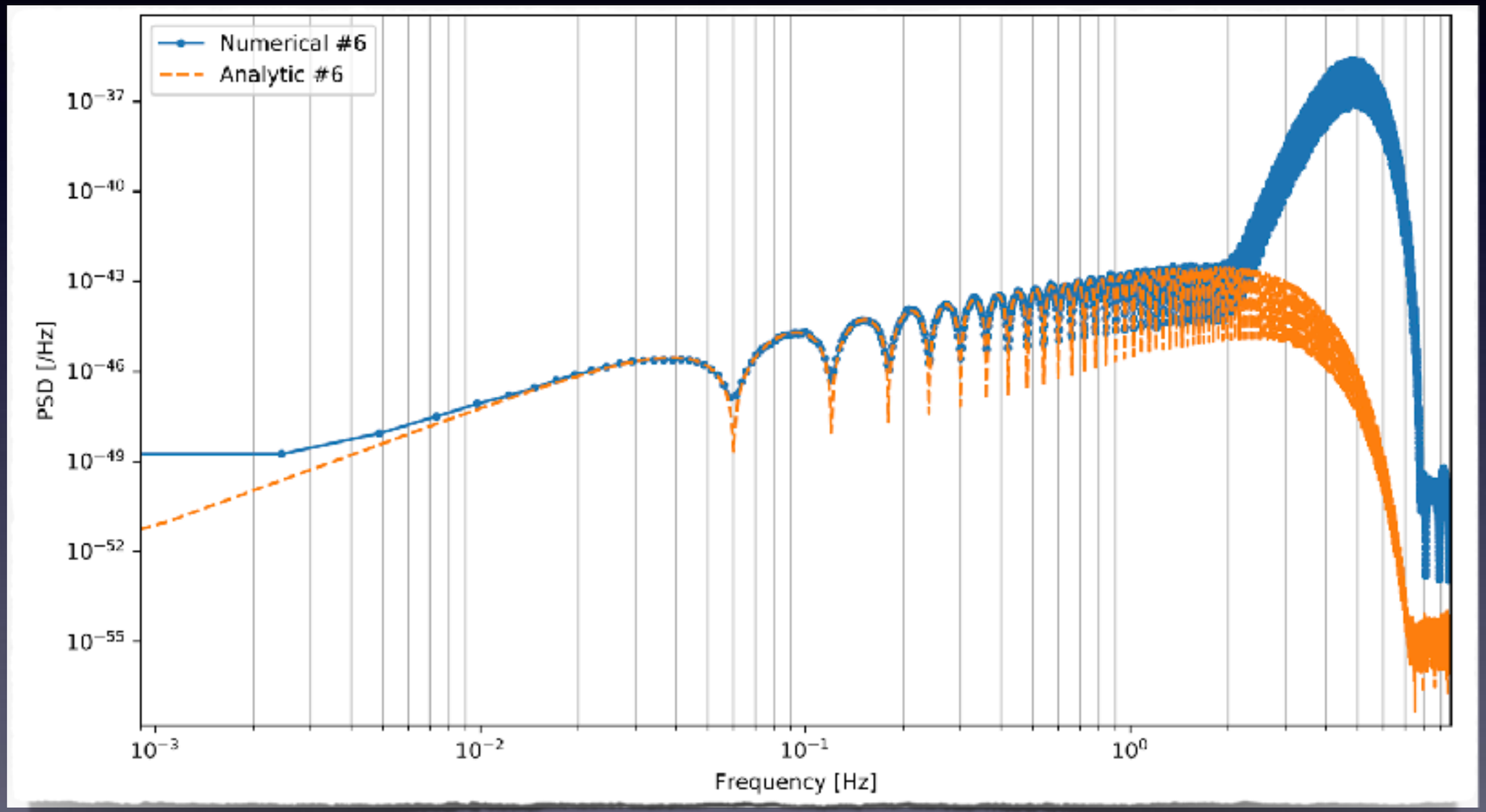
IMPACT ON LISA SCIENCE AND INSTRUMENT DESIGN

Modeling of Residuals



TDI 1.5 with linear orbits and no decimation

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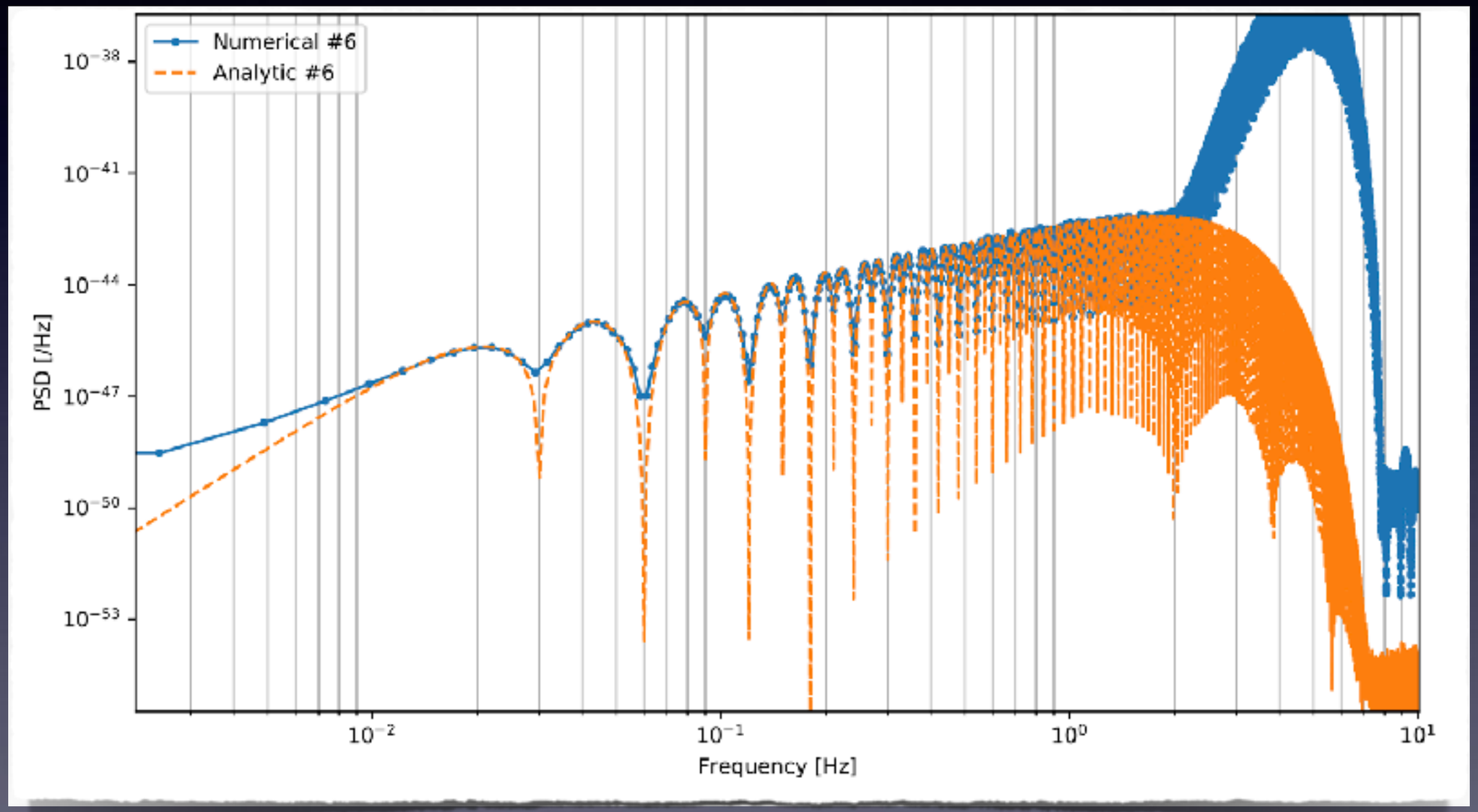
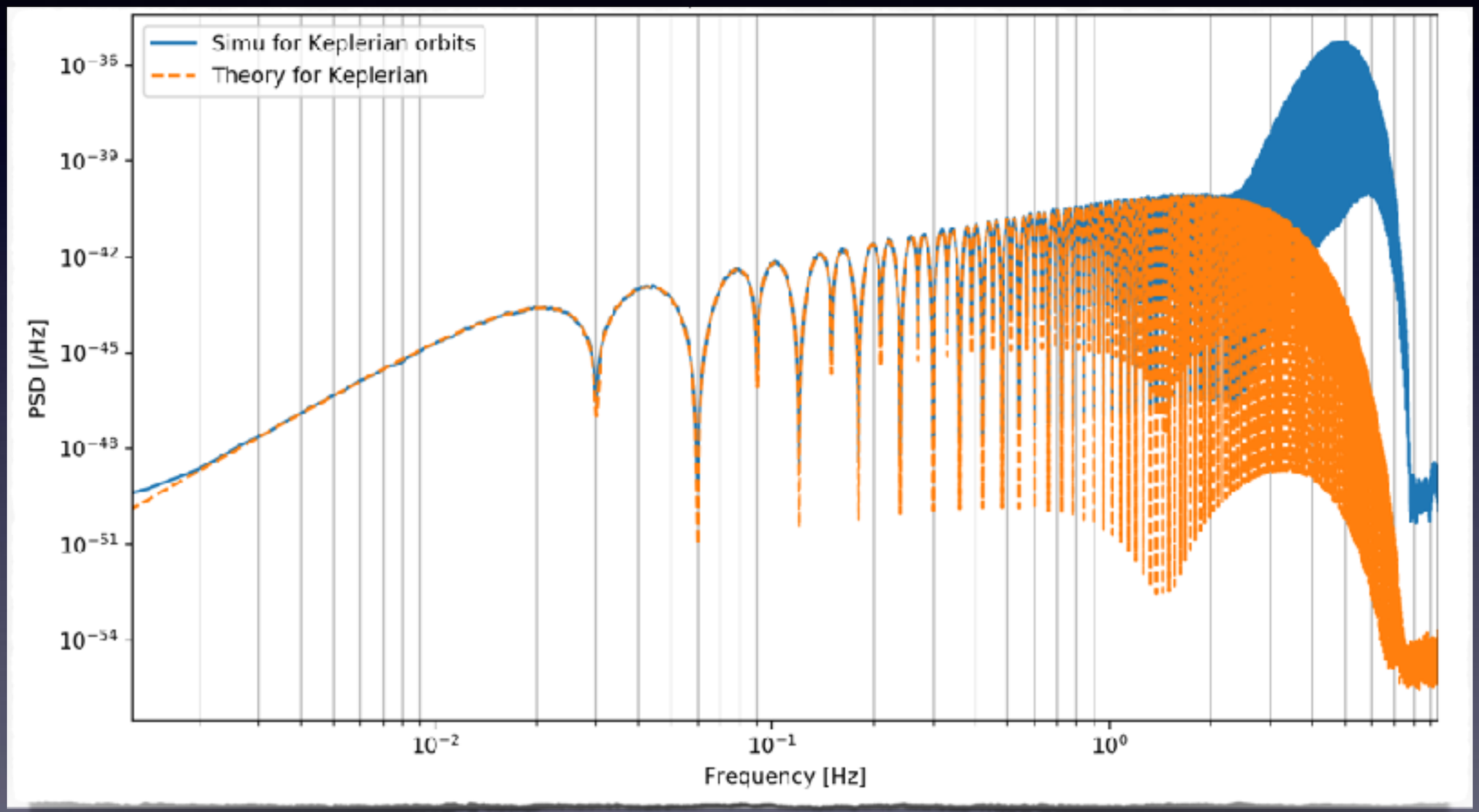


TDI 2 with linear orbits and no decimation

Modeling of Residuals



TDI 2 with realistic Keplerian orbits and no decimation

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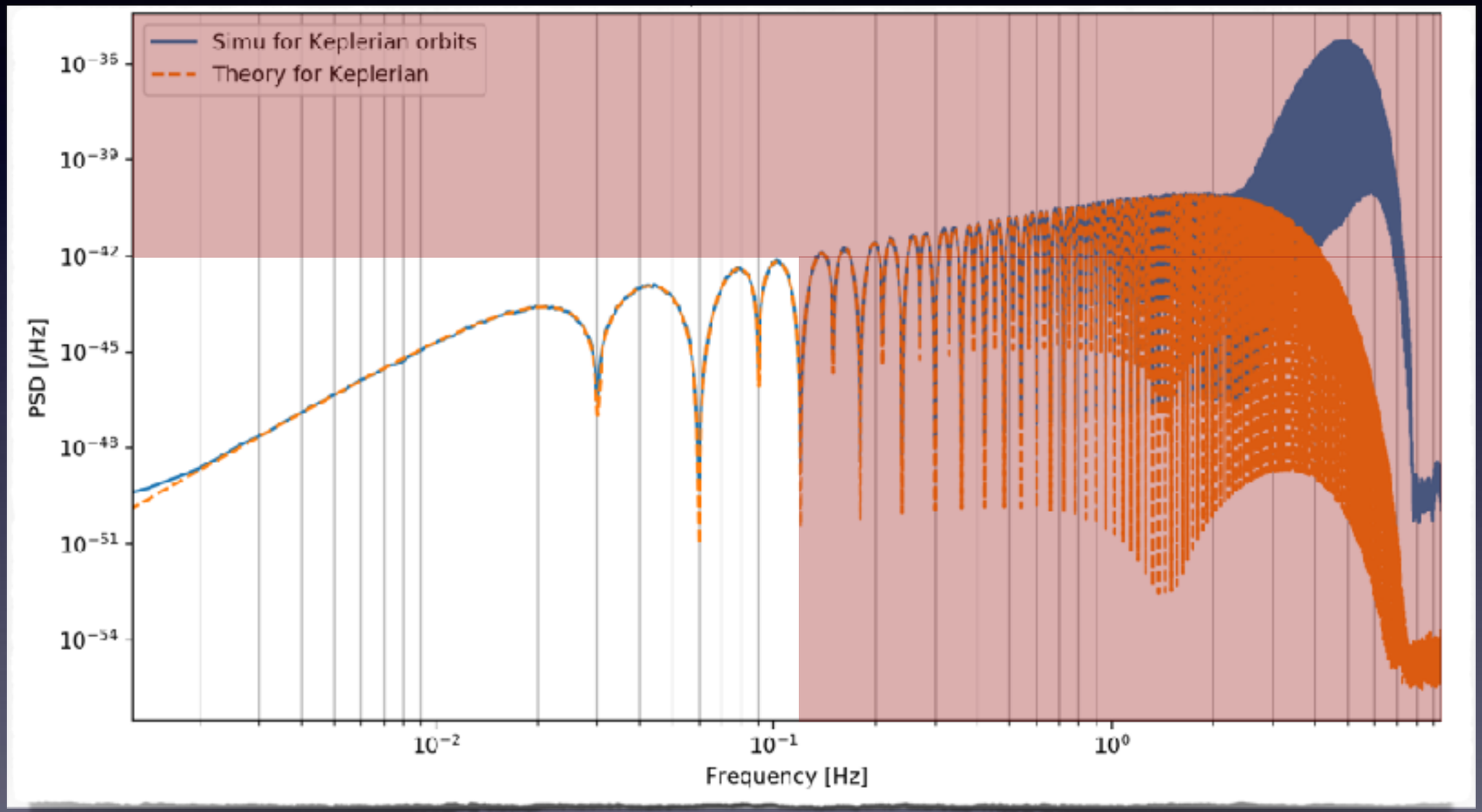

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    - Impacts on the global instrumental noise budget drive research and development, i.e. clocks, lasers, cross-talking systems

Impact on Science

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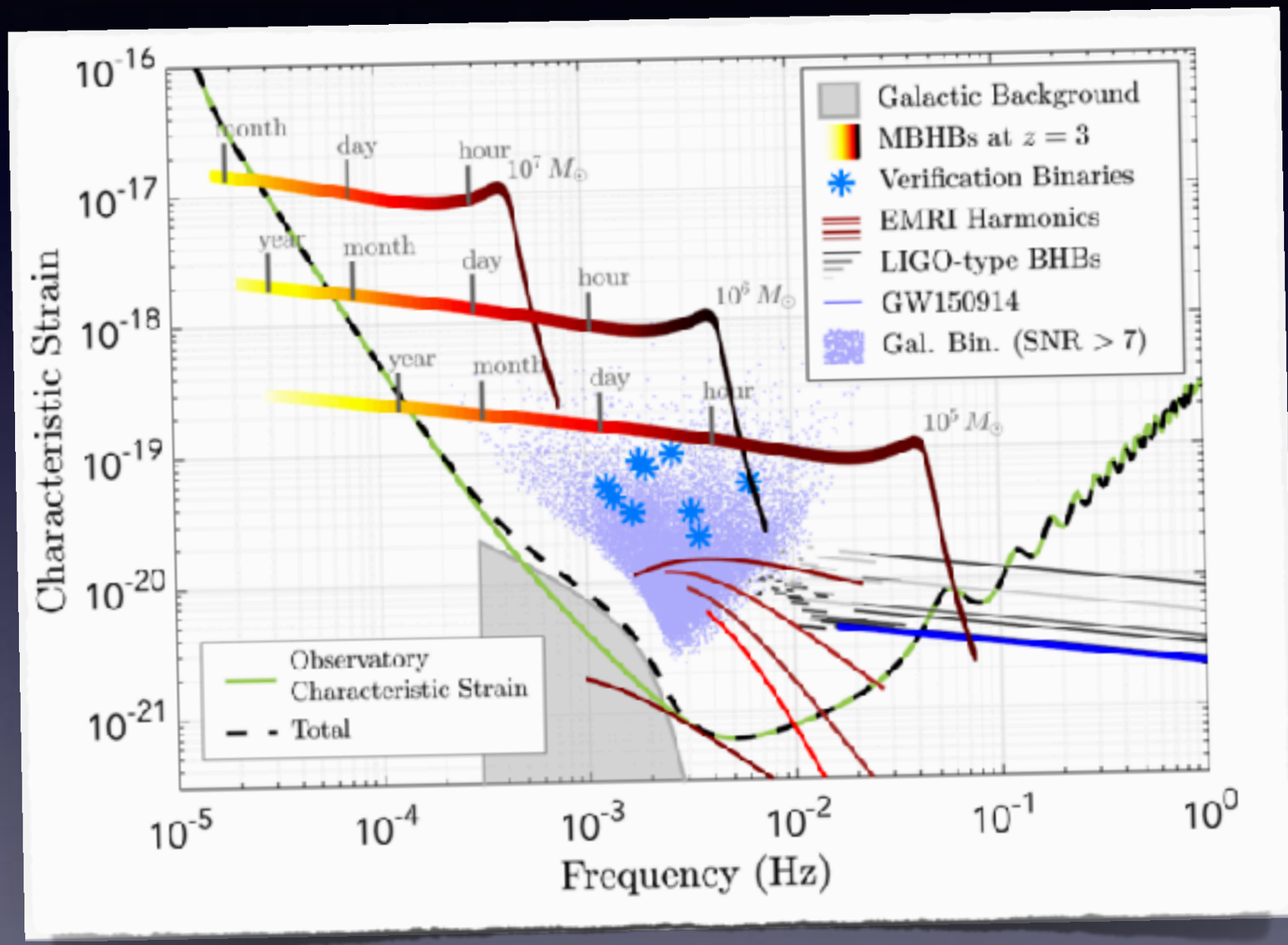
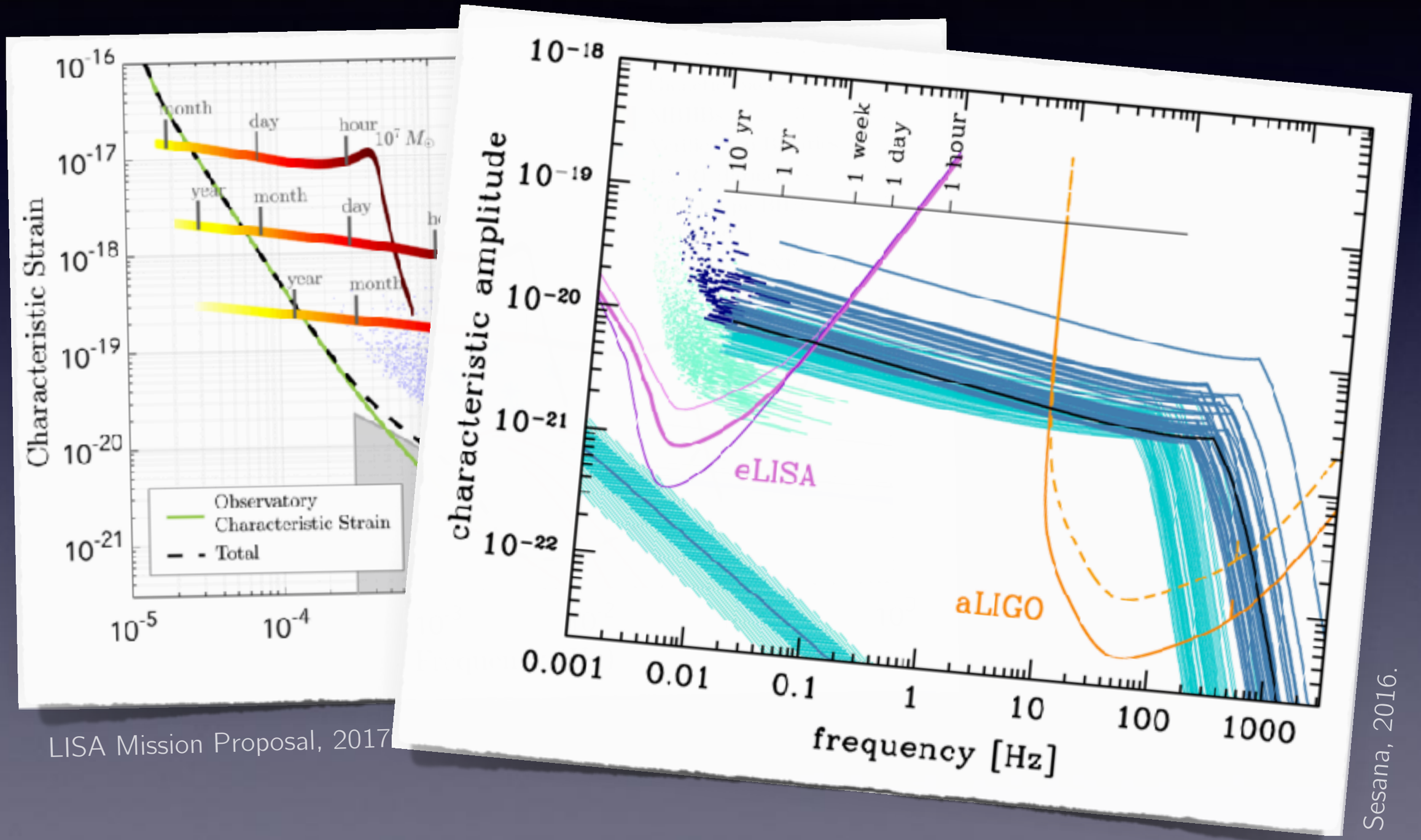


LISA Mission Proposal, 2017.

Impact on Science



LISA Mission Proposal, 2017

Sesana, 2016.

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