Pulsar glitches in full general relativity

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in collaboration with

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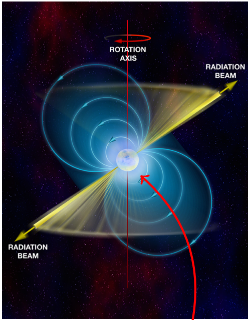
PSL RESEARCH UNIVERSITY PARIS

1. Introduction
   - Observations
   - Vortex-mediated glitch theory

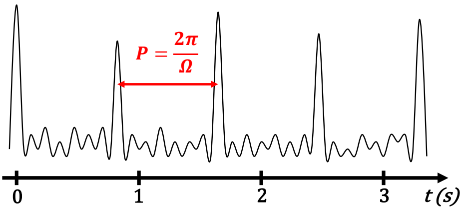
2. Simulations of pulsar glitches in GR
   - Realistic equilibrium configurations
   - Dynamics of giant glitches

3. Conclusion

The pulsar phenomenon

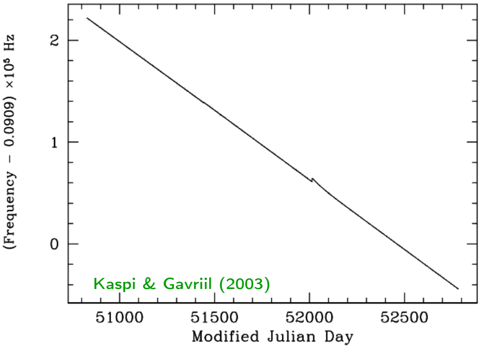


neutron star



The time evolution of P (or f) can be measured with a *very high precision*

The glitch phenomenon



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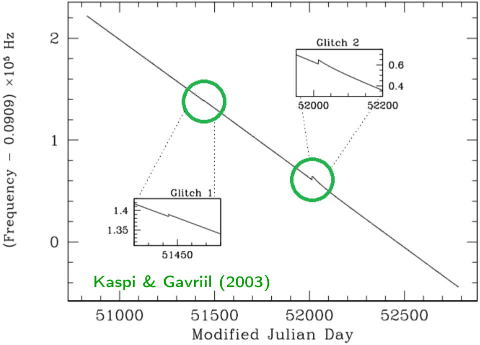

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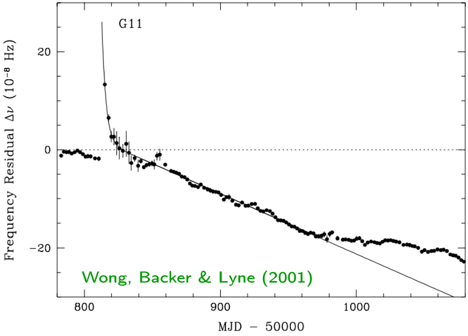


- **amplitude:**

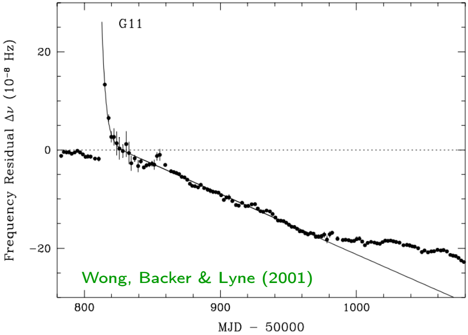
$$\Delta\Omega/\Omega \sim 10^{-11} - 10^{-5}$$

- short **rise time:**

$$\boxed{\tau_r < 30 \text{ s}} \quad \leftarrow\text{-- Vela}$$

- exponential **relaxation** on several days or months.

The glitch phenomenon



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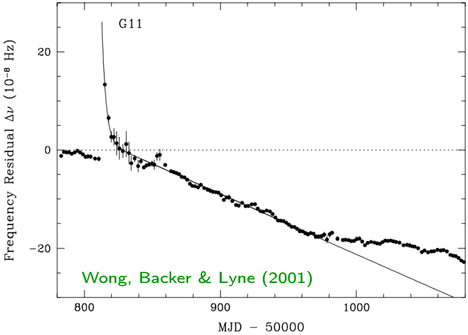
The glitch phenomenon



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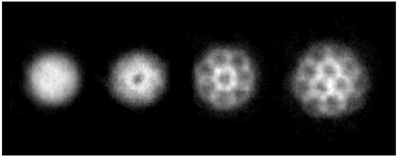
Angular momentum transfer between *two* fluids --→ **superfluidity**

Superfluidity in neutron stars

Superfluid properties:

- *null* viscosity,
- angular momentum carried by *quantized vortex lines*.



Madison *et al.* (2000)

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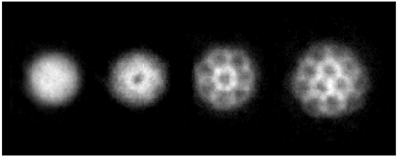

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Theoretical predictions

Critical temperature:

$$T_c^{\max} \simeq 10^9 - 10^{10}\ \mathrm{K}$$

⇢ superfluid neutrons in the core and in the inner crust

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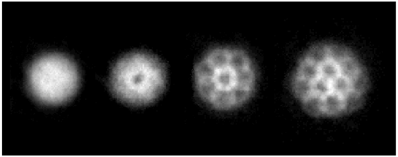

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Observational evidence

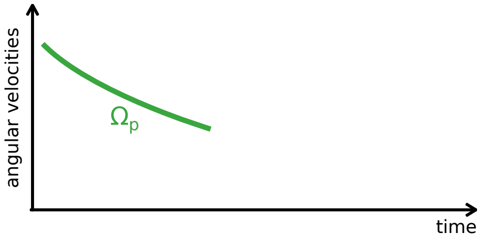
- *Long relaxation time scales in pulsar glitches,*
- Fast cooling of a young neutron star in Cassiopeia A, ...

Vortex-mediated glitch theory Anderson & Itoh (1975)

Two-fluid model

- Charged particles:

$$\Omega_p = \Omega \leftrightarrow \text{pulsar}$$



Vortex-mediated glitch theory
Anderson & Itoh (1975)

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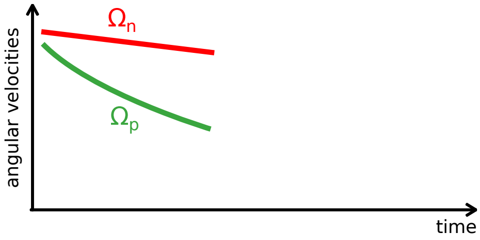
- Neutron superfluid:

$$\Omega_n \gtrsim \Omega_p$$



Key assumption:

$\longrightarrow$ vortices can pin to nuclei in the crust.

Vortex-mediated glitch theory Anderson & Itoh (1975)

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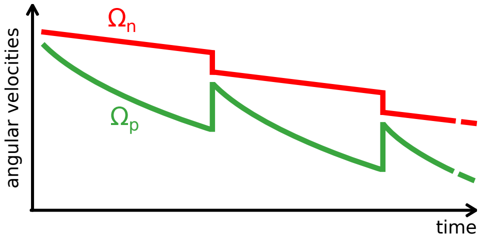

Once a critical lag $\delta\Omega = \Omega_n - \Omega_p$ is reached, some vortices get **unpinned** and are allowed to move **radially**.

$\dashrightarrow$ angular momentum transfer between the fluids = glitch!

This work

Question:

What is the impact of **general relativity** on the global dynamics of superfluid neutron stars during a glitch spin-up ?

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What is the impact of **general relativity** on the global dynamics of superfluid neutron stars during a glitch spin-up ?

→ *fundamental hypothesis*:

$$\tau_{\rm r} \gg \tau_{\rm h} \sim (G\bar{\rho})^{1/2} \simeq 0.1 \text{ ms}$$

a glitch event can be well described by a **quasi-stationary** sequence of **equilibrium** configurations

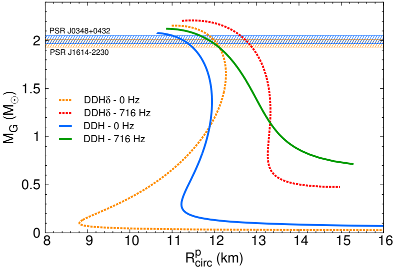
Assumptions & Ingredients
Prix *et al.* (2005) & Sourie *et al.* (2016)

Equilibrium configurations:

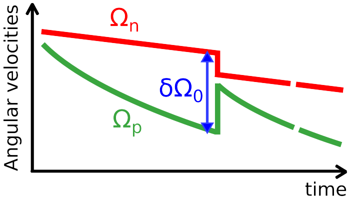
- **uniform** composition: n, p, e^-
  $\rightsquigarrow$ the crust is not considered,
- *stationary & axisymmetric* spacetime + isolated star,
- **rigid-body** rotation:
  $\rightsquigarrow$ Ω_n et Ω_p = const,
- $T \ll T_F$, no magnetic field,
- dissipative effects are neglected.

Equations of state:

- Polytropic EoSs,
- *Density-dependent RMF models* (DDH & DDHδ).



Angular momentum transfer Langlois *et al.* (1998)



$\Omega_n - \Omega_p = \delta\Omega_0 \rightsquigarrow$ the dynamics is governed by **mutual friction forces**

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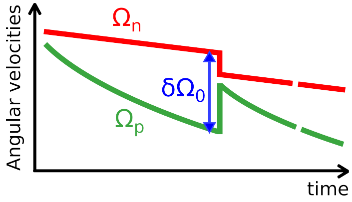

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Assuming *straight vortices*, the mutual friction moment considered reads

$$\Gamma_{mf} = -\bar{\mathcal{B}} \times \kappa \times (\Omega_n - \Omega_p)$$

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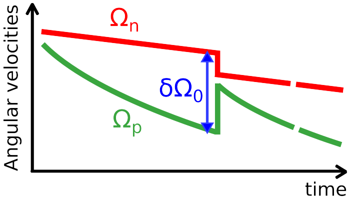

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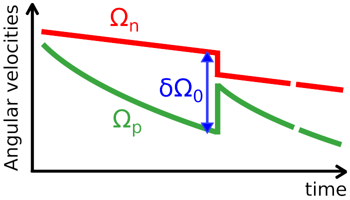

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mean mutual friction parameter

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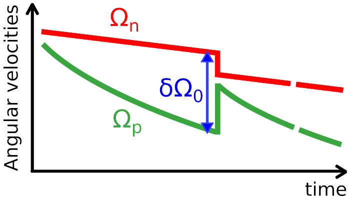

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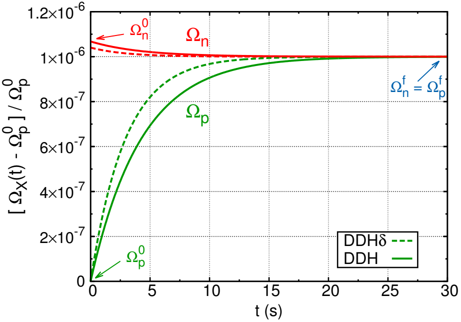
some prefactor

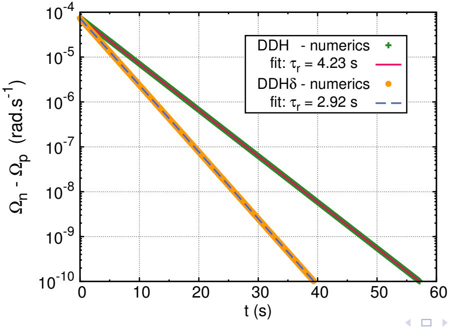
Time evolution

$$\begin{cases} \dot{J}_n = +\Gamma_{mf}, \\ \dot{J}_p = -\Gamma_{mf}. \end{cases}$$

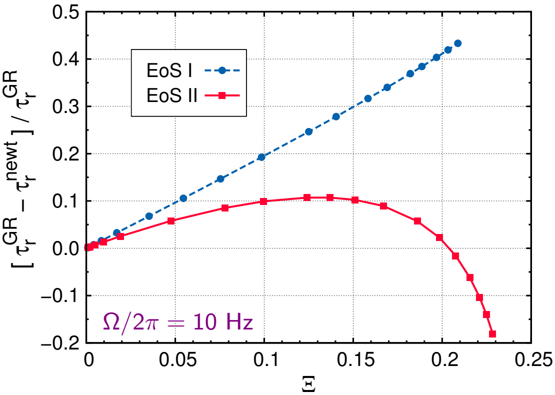
Computation of $\Omega_n(t)$ & $\Omega_p(t)$ profiles from $\Omega_{n,0} > \Omega_{p,0}$

$$\Delta\Omega/\Omega = 10^{-6},\ \Omega_n^f = \Omega_p^f = 2\pi \times 11.19\ \text{Hz},\ M_G = 1.4\ M_\odot\ \&\ \bar{\mathcal{B}} = 10^{-4}$$





Influence of general relativity on τ_r



- polytropic EoSs
- **compactness** parameter:

$$\Xi = \frac{G\, M_G}{R_{c,eq}\, c^2}$$

NB: for NSs, $\Xi \simeq 0.2$

- these relative differences also depend on Ω

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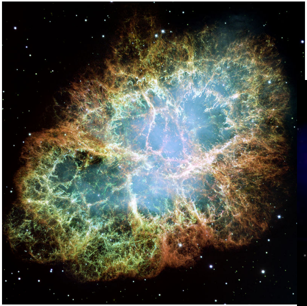
Conclusion & perspectives

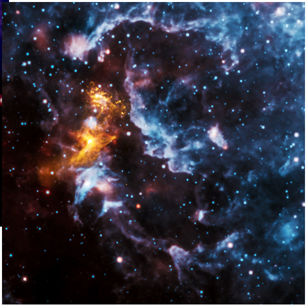
Relativistic corrections on the spin-up time: $\sim 50\%$,

$\hookrightarrow$ should be included in a quantitative model of glitches.

Future work:

- Improve our models to include the crust and to consider local glitch events,
- Compare with future accurate observations of glitches.

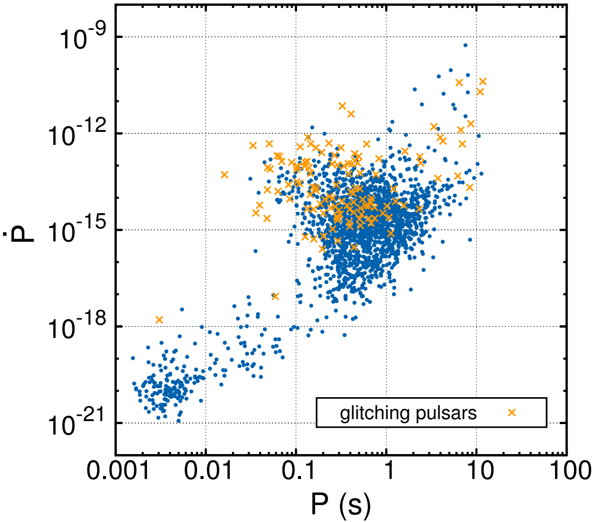




Thank you!

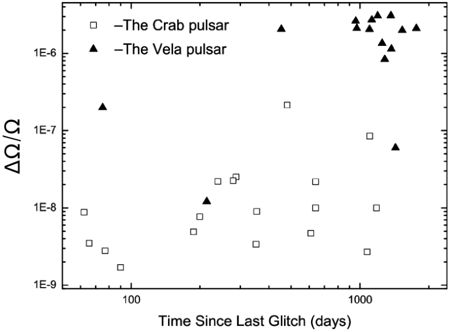
$P - \dot{P}$ diagram

ATNF Pulsar Database ; Manchester et al., *Astron. Journal*, 2005



Distinct glitching behaviors

Wang et al., *Ap&SS*, 2012



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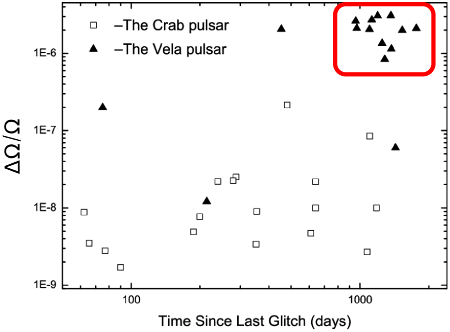

quasi-periodic giant glitches with a very narrow spread in size

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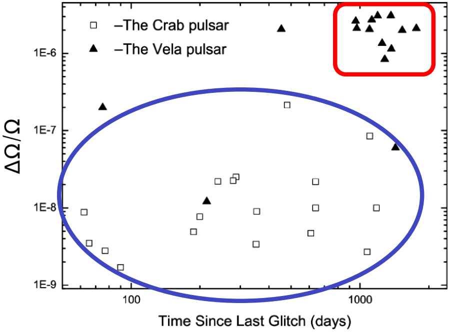

quasi-periodic giant glitches with a very narrow spread in size

glitches of various sizes at random intervals of time

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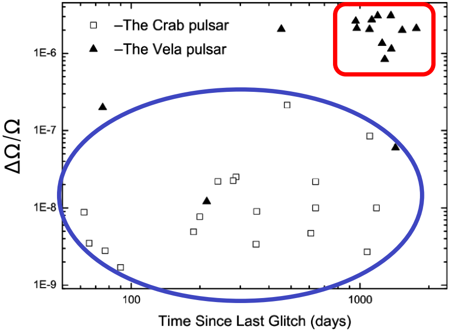

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glitches of various sizes at random intervals of time

Different models of glitches Haskell & Melatos, *IJMPD*, 2015

- Rearrangement of the moment of inertia ⇢ crustquakes,
- Angular momentum transfer between *two* fluids ⇢ **superfluidity**.

Spacetime metric

Bonazzola, Gourgoulhon, Salgado & Marck, *A&A*, 1993

Rotating neutron stars, at **equilibrium**, described by $(\mathcal{E}, \boldsymbol{g})$:

- asymptotically flat: $\boldsymbol{g} \to \boldsymbol{\eta}$ at spatial infinity ($r \to +\infty$),
- stationary & axisymmetric: $\frac{\partial g_{\alpha\beta}}{\partial t} = \frac{\partial g_{\alpha\beta}}{\partial \varphi} = 0$,
- circular: perfect fluids $\Rightarrow$ *purely circular* motion around the rotation axis with Ω_n, Ω_p (+ **rigid rotation**).

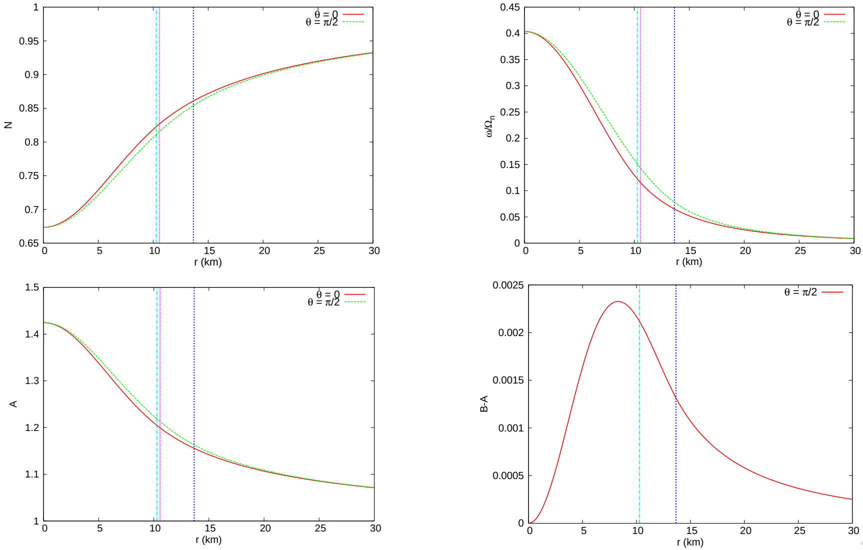
Spacetime metric in quasi-isotropic coordinates:

$$g_{\alpha\beta}\,\mathrm{d}x^\alpha\,\mathrm{d}x^\beta = -N^2\,\mathrm{d}t^2 + A^2(\mathrm{d}r^2 + r^2\,\mathrm{d}\theta^2) + B^2 r^2 \sin^2\theta(\mathrm{d}\varphi - \omega\,\mathrm{d}t)^2$$

At spatial infinity

$$N, A, B \to 1 \quad \& \quad \omega \to 0$$

Metric potentials



Relativistic two-fluid hydrodynamics

Carter, "Covariant theory of conductivity in ideal fluid or solid media", 1989 & Carter & Langlois, *Nuc. Phys. B*, 1998

System = two **perfect** fluids:

- superfluid neutrons $\rightarrow \vec{n}_{\rm n} = n_{\rm n}\vec{u}_{\rm n}$,
- protons & electrons $\rightarrow \vec{n}_{\rm p} = n_{\rm p}\vec{u}_{\rm p}$.

Energy-momentum tensor

$$T_{\alpha\beta} = n_{{\rm n}\alpha} p^{\rm n}_{\beta} + n_{{\rm p}\alpha} p^{\rm p}_{\beta} + \Psi g_{\alpha\beta}$$

$\hookrightarrow$ **conjugate momenta**

Entrainment matrix:

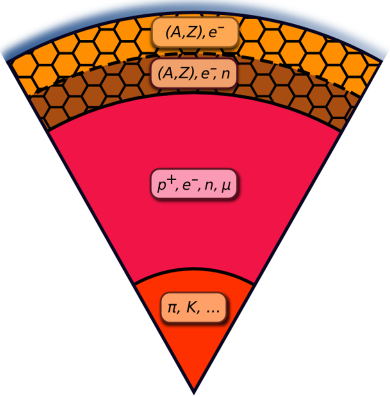
$$\begin{cases} p^{\rm n}_\alpha = \mathcal{K}^{\rm nn} n^{\rm n}_\alpha + \mathcal{K}^{\rm np} n^{\rm p}_\alpha \\ p^{\rm p}_\alpha = \mathcal{K}^{\rm pn} n^{\rm n}_\alpha + \mathcal{K}^{\rm pp} n^{\rm p}_\alpha \end{cases}$$

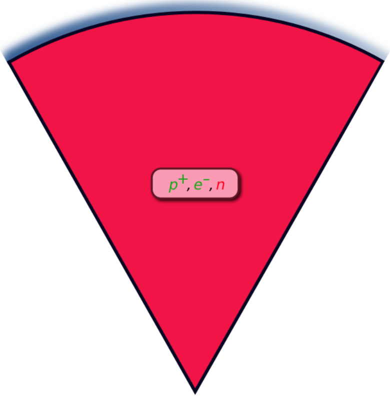
$\dashrightarrow$ entrainment effect

Equation of state

$$\mathcal{E}(n_{\rm n}, n_{\rm p}, \Delta^2)$$

Neutron stars interior

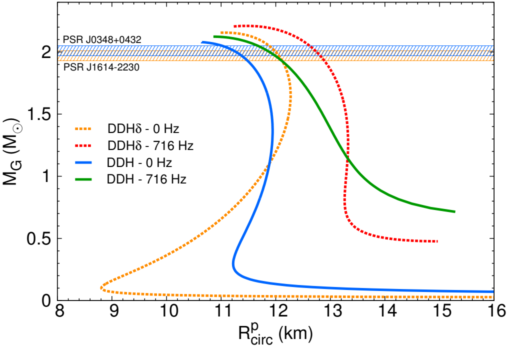




Equations of state

Relativistic Mean-Field Theory:

strong interaction between nucleons ⇔ exchange of effective mesons



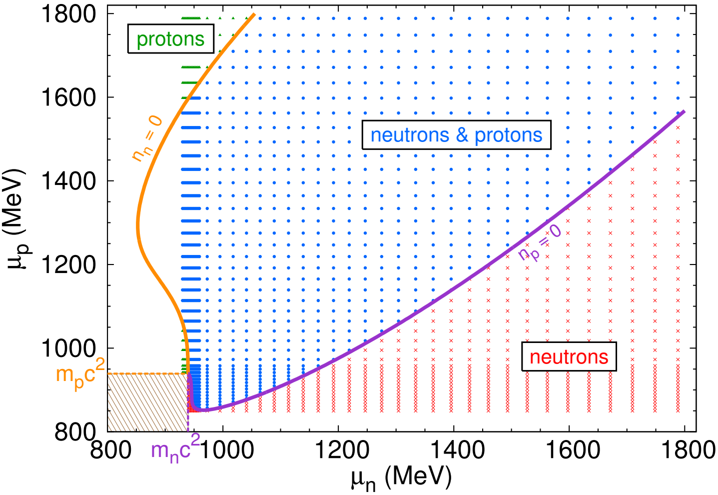
- Gravitational mass:

$$M_G = M^B + E_{bind},$$

- Circumferential radius:

$$R^X_{circ,\ eq} = \mathcal{C}^X / 2\pi.$$

Tabulated EoS



Entrainment effects

Dynamical effective mass:

$$\boxed{{}^3\vec{\boldsymbol{p}}_{\boldsymbol{X}} = m^*_X \; {}^3\vec{\boldsymbol{u}}_{\boldsymbol{X}}}$$

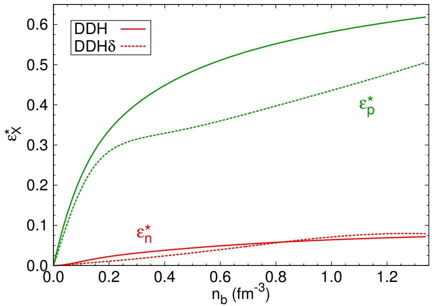
$\rightarrow$ in the *rest frame* of the second fluid.

Zero-velocity frame:

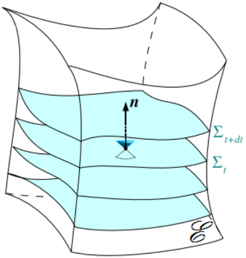
special relativity

$$m^*_X = \mu^X \times \left(1 - \varepsilon^*_X\right)$$

entrainment

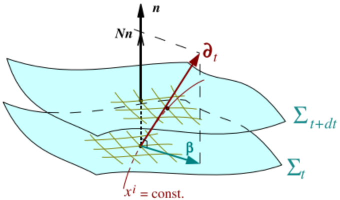


3+1 formalism



Foliation of the spacetime $(\mathcal{E}, \boldsymbol{g})$ by $(\Sigma_t)_{t\in\mathbb{R}}$, with unit normal $\vec{\boldsymbol{n}}$

Eulerian observer $\mathcal{O}_n$: 4-velocity $= \vec{\boldsymbol{n}}$



- lapse function N : $\vec{\boldsymbol{n}} = -N\vec{\boldsymbol{\nabla}}t$,
- shift vector $\vec{\beta}$: $\vec{\boldsymbol{\partial}}_t = N\vec{\boldsymbol{n}} + \vec{\beta}$.

3+1 metric:

$$g_{\alpha\beta}\,\mathrm{d}x^\alpha\,\mathrm{d}x^\beta = -N^2\,\mathrm{d}t^2 + \gamma_{ij}\left(\mathrm{d}x^i + \beta^i\,\mathrm{d}t\right)\left(\mathrm{d}x^j + \beta^j\,\mathrm{d}t\right)$$

Numerical procedure

Paramètres d'entrée :

- une EOS
- H_c^n, H_c^p
- Ω_n, Ω_p

$i = 0$

Initialisation :

- $N = A = B = 1$ et $\omega = 0, \forall\,(r, \theta)$
- $U_n = U_p = 0$
- $H_0^i(r,\theta) = H_c^i\left(1 - \frac{r^2}{R^2}\right)$

Convergence threshold

$$|H_{k+1}^i(r,\theta) - H_k^i(r,\theta)| < \epsilon$$

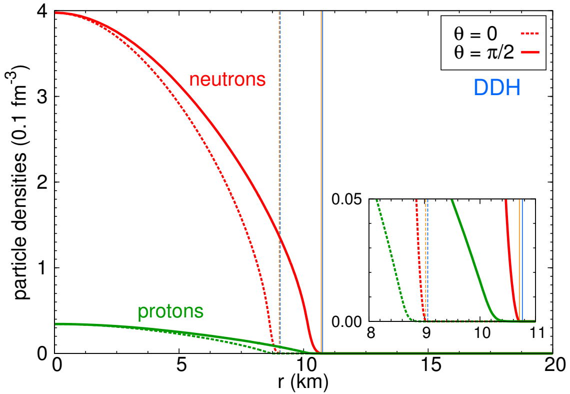
At each iteration

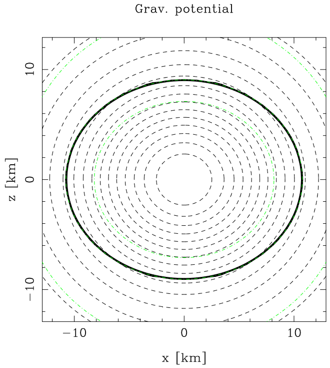
For given values of $(\mu^{\mathrm{n}}, \mu^{\mathrm{p}}, \Delta^2)$, we compute:

1. Ψ, n_{n}, n_{p} and α from the EoS
2. The source terms E, p_φ, ${S^i}_i$,
3. Einstein Equations are solved,
4. Kinetic terms U_{i} et Γ_{i},
5. Computation of H_{k+1}^i.

Density profiles

$$M_G = 1.4\ M_\odot,\ \Omega_n/2\pi = \Omega_p/2\pi = 716\ \text{Hz}$$

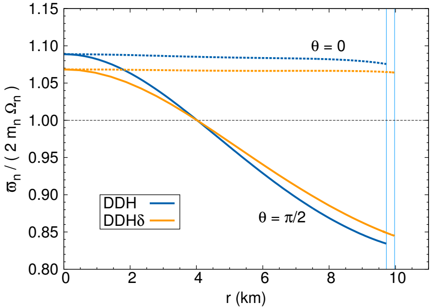


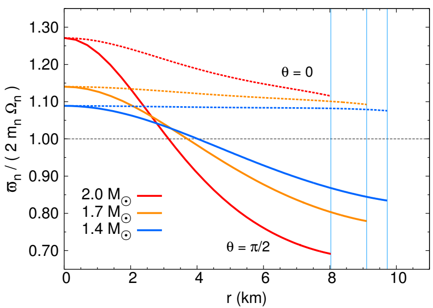


Vorticity

Superfluid vorticity

$$w_{\mu\nu} = \nabla_\mu p^n_\nu - \nabla_\nu p^n_\mu \dashrightarrow \varpi_n = \sqrt{\frac{w_{\mu\nu} w^{\mu\nu}}{2}}$$





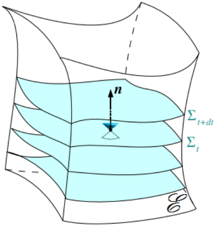
$$\Omega^n/2\pi = \Omega^p/2\pi = 716 \text{ Hz}$$

Angular momenta

Axisymmetry $\leftrightarrow \vec{\chi}$

Komar definition:

$$J_K = -\int_{\Sigma_t} \underbrace{\boldsymbol{T}(\vec{\boldsymbol{n}}, \vec{\chi})}_{-p_\varphi} \, \mathrm{d}^3 V$$



Eulerian observer $\vec{\boldsymbol{n}}$ (3+1)

Angular momentum of each fluid

Langlois, Sedrakian & Carter, *MNRAS*, 1998

$$p_\varphi = \underbrace{\Gamma_n n_n p^n_\varphi}_{j^n_\varphi} + \underbrace{\Gamma_p n_p p^p_\varphi}_{j^p_\varphi}$$

$$J_X = \int_{\Sigma_t} j^X_\varphi \, A^2 B r^2 \sin\theta \, \mathrm{d}r \, \mathrm{d}\theta \, \mathrm{d}\varphi$$

Fluid couplings

In the slow-rotation approximation and to first order in the lag $\delta\Omega = \Omega_n - \Omega_p$, the **angular momentum of fluid** X reads

$$J_X \simeq \int_{\Sigma_t} n_X \mu^X \frac{B}{N} (\Omega_X - \omega) r^2 \sin^2\theta \, d^3V + \int_{\Sigma_t} n_X \mu^X \varepsilon_X \frac{B}{N} (\Omega_Y - \Omega_X) r^2 \sin^2\theta \, d^3V$$

Introducing $i_X \equiv n_X \mu^X \frac{B}{N} r^2 \sin^2\theta$, we characterize the couplings by

- Entrainment:
- Lense-Thirring:

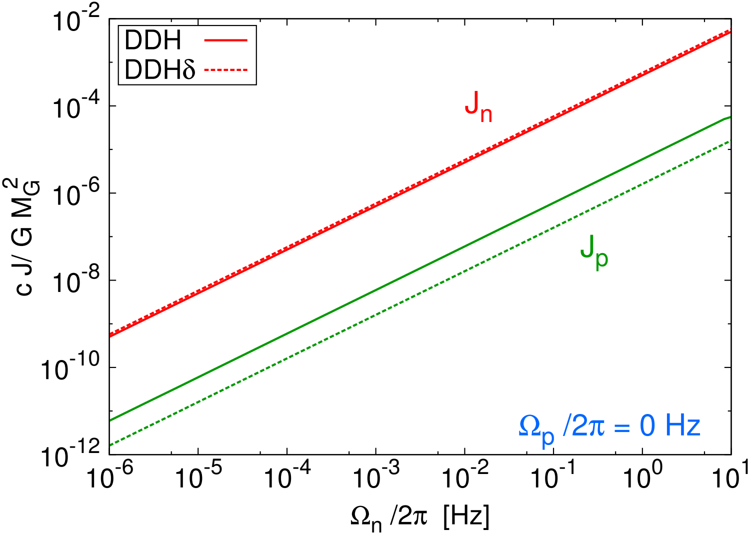
$$\tilde{I}_X \, \tilde{\varepsilon}_X \equiv \int_{\Sigma_t} i_X \, \varepsilon_X \, d^3V$$

$$\tilde{I}_X \left(\varepsilon^{LT}_{X\to X}\Omega_X + \varepsilon^{LT}_{Y\to X}\Omega_Y\right) \equiv \int_{\Sigma_t} i_X \, \omega \, d^3V$$

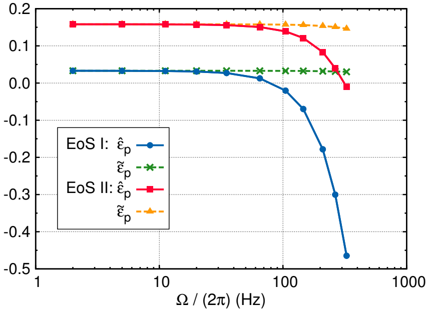
where $\tilde{I}_X \equiv \int_{\Sigma_t} i_X \, d^3V$

$$J_X = \tilde{I}_X \left(1 - \varepsilon^{LT}_{X\to X} - \tilde{\varepsilon}_X\right) \Omega_X + \tilde{I}_X \left(\tilde{\varepsilon}_X - \varepsilon^{LT}_{Y\to X}\right) \Omega_Y$$

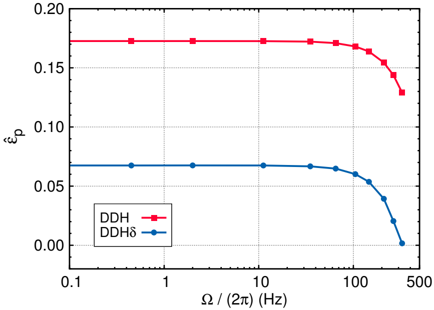
Fluid couplings



Influence of Ω on the couplings



Newtonian gravity



general relativity

Fluid couplings

Moments of inertia:

$$dJ_X = I_{XX}\ d\Omega_X + I_{XY}\ d\Omega_Y \qquad X, Y \in \{n, p\}$$

$$\hat{I}_X = I_{XX} + I_{XY} \qquad \hat{I} = \hat{I}_n + \hat{I}_p$$

Fluid couplings

Moments of inertia:

$$dJ_X = I_{XX}\, d\Omega_X + I_{XY}\, d\Omega_Y \qquad X, Y \in \{n, p\}$$

$$\hat{I}_X = I_{XX} + I_{XY} \qquad \hat{I} = \hat{I}_n + \hat{I}_p$$

In the slow-rotation approximation ($\Omega_n, \Omega_p \ll \Omega_K$), the fluids are mainly coupled through two *non-dissipative* mechanisms:

- entrainment effect

  due to the strong interactions between nucleons *in the core*:

  $$p^\alpha_X = \mathcal{K}^{XX} n_X u^\alpha_X + \mathcal{K}^{XY} n_Y u^\alpha_Y$$

  Andreev & Bashkin, *SJETP*, 1976

- relativistic frame-dragging effect

  associated with the rotation of the two fluids, Ω_n and Ω_p:

  $$g_{t\varphi} \neq 0$$

  Carter, *Annals of Physics*, 1975

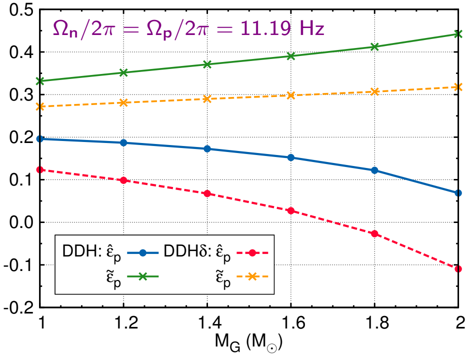
Entrainment VS frame-dragging

Coupling coefficients:

$$\hat{\varepsilon}_X = I_{XY} / \hat{I}_X$$

In the slow-rotation approximation:

$$\hat{\varepsilon}_{\rm p} = \frac{\tilde{\varepsilon}_{\rm p} - \varepsilon^{LT}_{\rm n\to p}}{1 - \varepsilon^{LT}_{\rm p\to p} - \varepsilon^{LT}_{\rm n\to p}}$$



Remarks:

- $\tilde{\varepsilon}_X$ characterizes entrainment,
- in Newtonian gravity:

$$\hat{\varepsilon}_X = \tilde{\varepsilon}_X$$

NB: $\hat{\varepsilon}_{\rm n} = \hat{I}_{\rm p}/\hat{I}_{\rm n} \times \hat{\varepsilon}_{\rm p} \simeq 0.05 \times \hat{\varepsilon}_{\rm p}$

Where does the vortex unpinning take place?

Glitches have been generally thought to originate from the crust, because:

- the core superfluid was expected to be strongly coupled to the crust Alpar et al., *ApJ*, 1984
- the analysis of glitch data suggested that the superfluid represents a few percent of the total angular momentum of the star Link et al., *PRL*, 1999

However, this scenario has been recently **challenged**:

- considering entrainment effects, the crust does not carry enough angular momentum Andersson et al., *PRL*, 2012 & Chamel, *PRL*, 2013
- a huge glitch has been observed in PSR 2334+61 Alpar, *AIP Conf.Proc.*, 2011
- the core superfluid could be decoupled from the rest of the star, if vortices are pinned to flux tubes Gügercinoglu & Alpar, *ApJ*, 2014

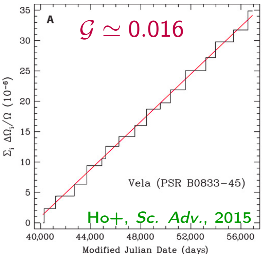
The core superfluid plays a more important role than previously thought.

Additional physical inputs

So far, we assumed that **all** the neutrons can decouple from the protons.

- only a *small fraction* of the neutron fluid could be involved in the glitch:

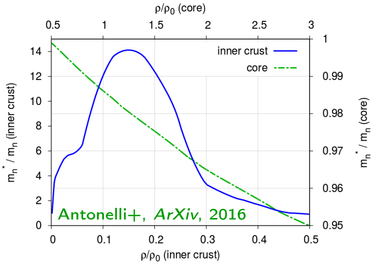
$$\bar{I}_n/\bar{I} > f \equiv I_n^{nc}/\bar{I} \gtrsim \mathcal{G} \times (1 - \varepsilon_n^{nc})$$



See also: Link+, *PRL*, 1999 & Lyne+, *MNRAS*, 2000

- we need to account for *crustal entrainment* (Bragg scattering):

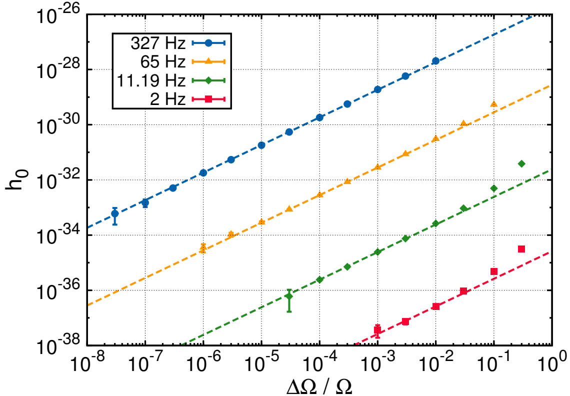
$$-14 \lesssim \varepsilon_n^{nc} \lesssim 0$$



See also: Chamel, *PRC*, 2012

Gravitational wave amplitude

$$h_+(t) = -\frac{3}{2}\sin^2 i \, \frac{G}{Dc^4}\ddot{Q} = h_0 \sin^2 i \; e^{-\frac{t}{\tau_r}}$$

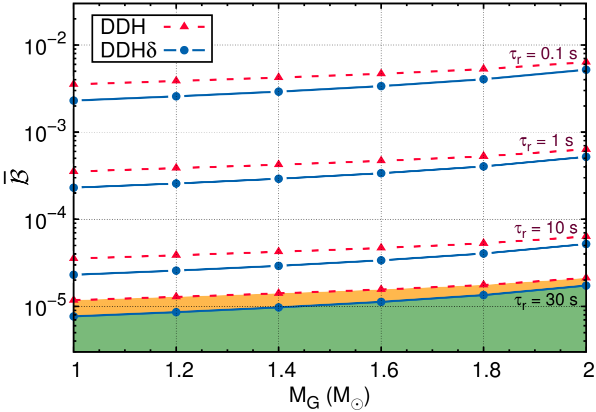


- $D = 1$ kpc,
- $\bar{\mathcal{B}} = 10^{-3}$,
- $M_G = 1.4\ M_\odot$,
- DDH EoS.

$$h_0 \simeq 1.0 \times 10^{-37} \left(\frac{D}{1\ \text{kpc}}\right)^{-1} \left(\frac{\bar{\mathcal{B}}}{10^{-3}}\right)^{2} \left(\frac{\Omega}{10^2\ \text{rad.s}^{-1}}\right)^{4} \left(\frac{\Delta\Omega/\Omega}{10^{-6}}\right)$$

The Vela pulsar

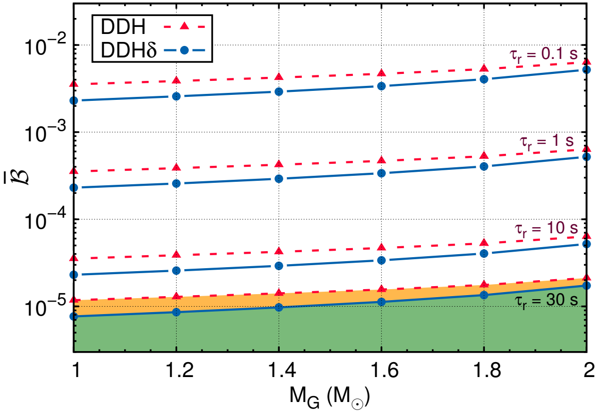
$$\Delta\Omega/\Omega = 10^{-6},\ \Omega_n^f = \Omega_p^f = 2\pi \times 11.19\ \text{Hz}$$



- $\bar{\mathcal{B}} \nearrow \implies \tau_r \searrow$

The Vela pulsar

$\Delta\Omega/\Omega = 10^{-6},\ \Omega_n^f = \Omega_p^f = 2\pi \times 11.19$ Hz

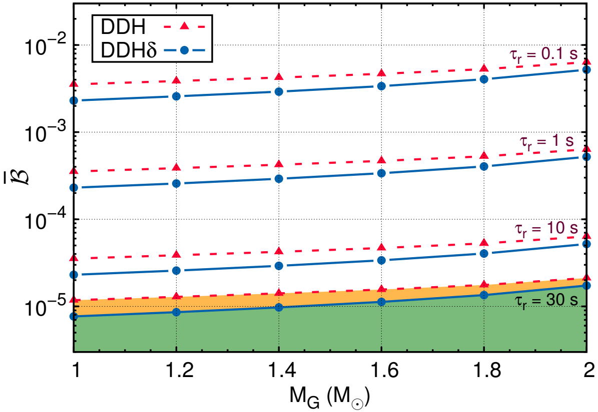


- $\bar{\mathcal{B}} \nearrow \implies \tau_r \searrow$
- *Constraint on* $\bar{\mathcal{B}}$:

  $\tau_r < 30$ s $\Rightarrow \bar{\mathcal{B}} > 10^{-5}$

The Vela pulsar

$\Delta\Omega/\Omega = 10^{-6}$, $\Omega_n^f = \Omega_p^f = 2\pi \times 11.19$ Hz



- $\bar{\mathcal{B}} \nearrow \Longrightarrow \tau_r \searrow$
- *Constraint on* $\bar{\mathcal{B}}$:

  $\tau_r < 30$ s $\Rightarrow \bar{\mathcal{B}} > 10^{-5}$
- $\bar{\mathcal{B}} < 0.5 \rightsquigarrow \tau_r > 0.6$ ms

$\hookrightarrow$ the glitch event is a **quasi-stationary** process