Photons at rest around boson stars

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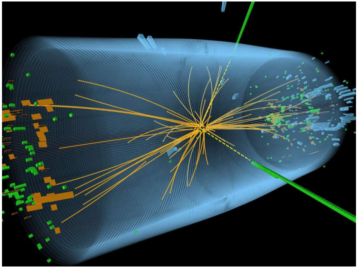
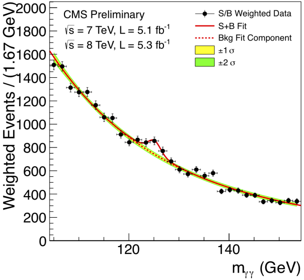
What is a boson star ?

- Localized configuration of a self-gravitating complex scalar field.
- Introduced in the 1960s by Bonazzola, Pacini, Kaup and Ruffini.
- Corresponds to spin-0 particle $\Longrightarrow$ *boson*.
- At least one scalar field in nature :*Higgs boson*.





Boson star model

- Scalar field has a $U(1)$ symmetry:

$$\Phi \longrightarrow \Phi \exp(i\alpha)\,.$$

- The Lagrangian of the field is

$$\mathcal{L}_M = -\frac{1}{2}\left[g^{\mu\nu}\nabla_\mu\bar{\Phi}\nabla_\nu\Phi + V\left(|\Phi|^2\right)\right].$$

  V is a potential (for a free field $V = m^2/\hbar^2\,|\Phi|^2$)
- The Lagrangian of gravity is

$$\mathcal{L}_g = \frac{1}{16\pi}R$$

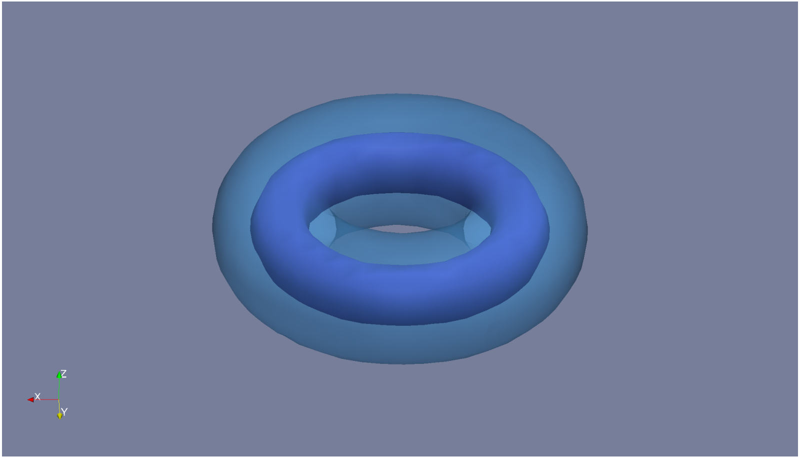
The variation of the equation gives the Einstein-Klein-Gordon system.

Ansatz for the field

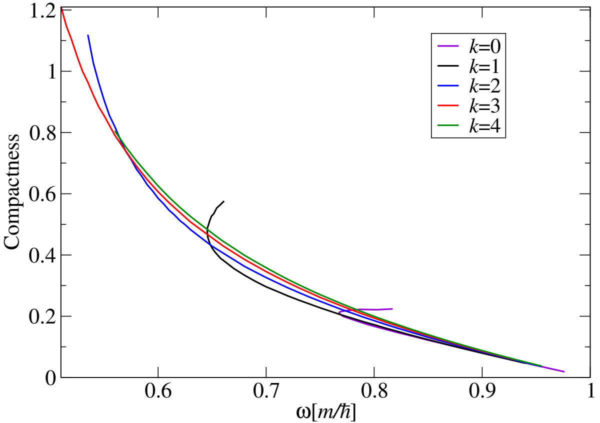
$\Phi = \phi \exp\left[i\left(\omega t - k\varphi\right)\right]$

- $U\left(1\right) \Longrightarrow$ the action does not depend on $\left(t,\varphi\right)$.
- ϕ and the metric fields depend only on $\left(r,\theta\right)$.
- k and ω appear as parameters in the equations.
- k is an integer and ω a real number.
- $k=0$ corresponds to spherically symmetric configurations.

Field configuration ; $k = 2$



Compactness



Effective potential method

- Metric in quasi-isotropic coordinates :

$$g_{\mu\nu}\mathrm{d}x^{\mu}\mathrm{d}x^{\nu} = -N^2\mathrm{d}t^2 + A^2\left(\mathrm{d}r^2 + r^2\mathrm{d}\theta^2\right) + B^2 r^2 \sin^2\theta \left(\mathrm{d}\varphi + \beta^{\varphi}\mathrm{d}t\right)^2$$

- Light rings : closed circular orbits of photons.

$$U^{\alpha} = \left(U^t, U^r, 0, U^{\varphi}\right)$$

- Two conserved quantities $U_{\alpha}\left(\partial_t\right)^{\alpha} = -E$ and $U_{\alpha}\left(\partial_{\varphi}\right)^{\alpha} = L$.
- Null geodesics $U_{\alpha}U^{\alpha} = 0$ leads to $\left(U^r\right)^2 + V_{\text{eff}}\left(r\right) = 0$

$$V_{\text{eff}} = \frac{1}{A^2}\left[-\frac{\left(\beta^{\varphi}L + E\right)^2}{N^2} + \frac{L^2}{B^2 r^2}\right]$$

Circular orbits

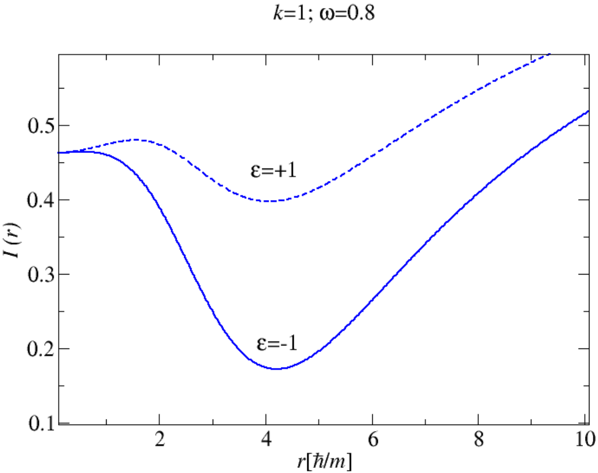
- Circular orbits require $V_{\text{eff}} = 0$ and $\partial_r V_{\text{eff}} = 0$
- First condition implies

$$\frac{E}{L} = -\beta^{\varphi} + \epsilon \frac{N}{Br} \quad ; \quad \epsilon = \pm 1$$

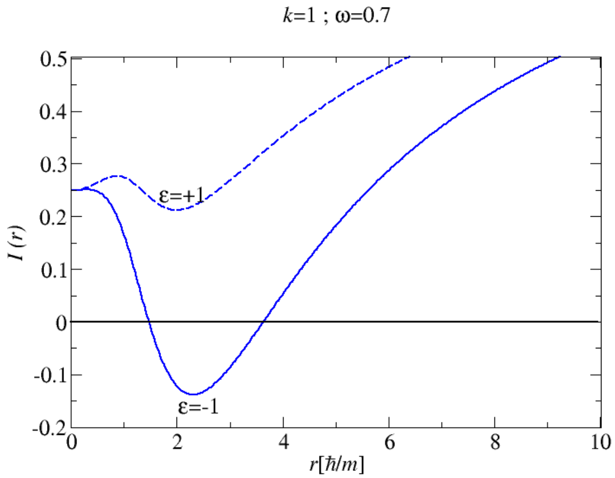
- Contrary to the massive case only the ratio E/L is constrained.
- Second condition reduces to finding the zeros of

$$I(r) = \left(\epsilon \frac{\partial_r \beta^{\varphi}}{NB} \right) r^2 + \left(\frac{\partial_r B}{B^3} - \frac{\partial_r N}{NB^2} \right) r + \frac{1}{B^2}$$

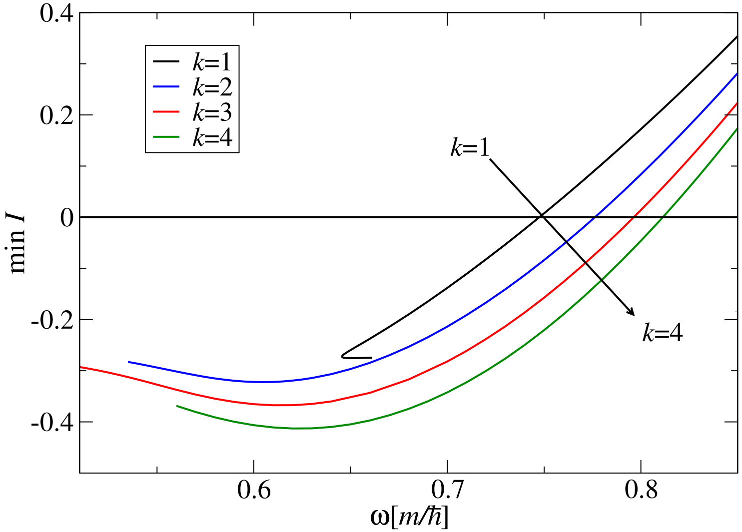
$I(r)$ without light rings



$I(r)$ with light rings



Existence of light rings



Properties

The orbital frequency is given by $\frac{\mathrm{d}\varphi}{\mathrm{d}t} = -\frac{N}{Br} - \beta^{\varphi}$.

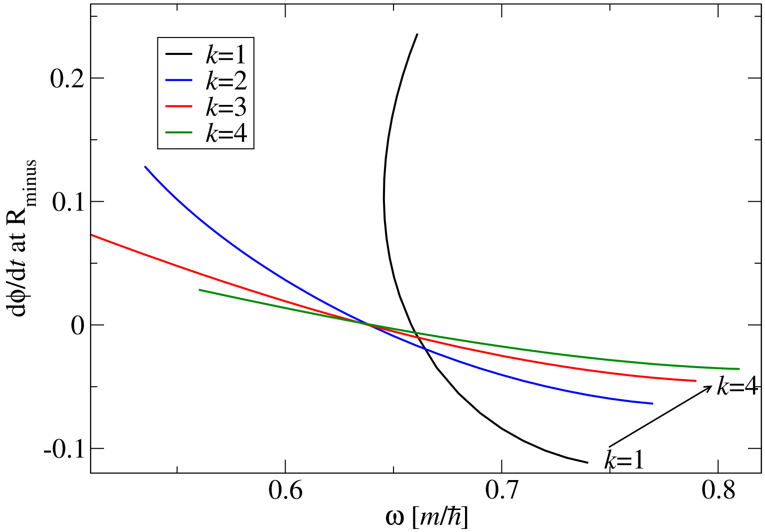
Boson star case

- Two light rights for relativistic enough configurations.
- The outer one is unstable and retrograde.
- The outer one is stable (and changes type).

Black hole case

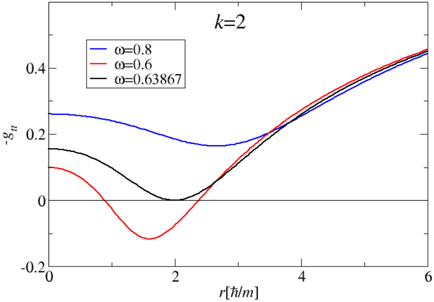
- Two light rings for Kerr black holes.
- One prograde and one retrograde.
- Both unstable.

Orbital frequency of the inner light ring



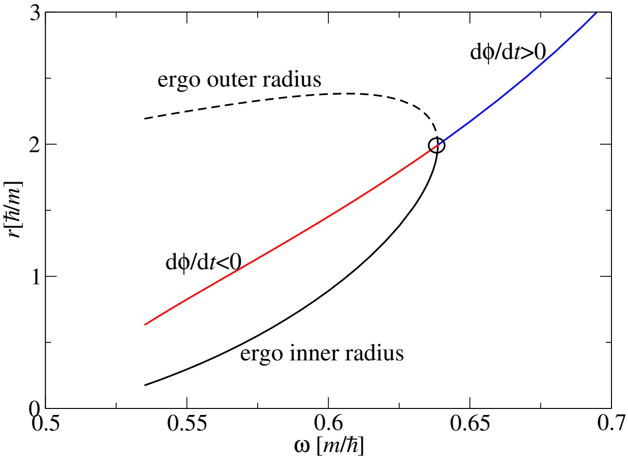
Light points

- On each sequence : a photon **at rest** : $U^{\mu} = (1, 0, 0, 0)$.
- Null condition : $U_{\mu}U^{\mu} = 0 \Longrightarrow g_{tt} = 0$. The light point lies exactly on the boundary of an ergoregion.
- Additional condition : geodesic equation $\Longrightarrow \partial_r g_{tt} = 0$

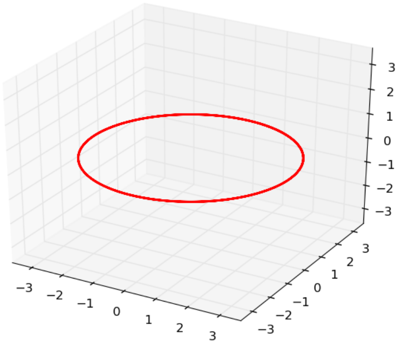


Light points and ergoregions

The light point lie where an ergoregion just starts to develop

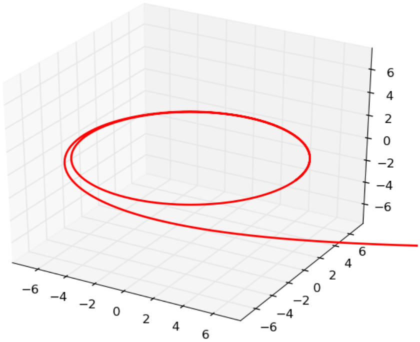


Inner light ring



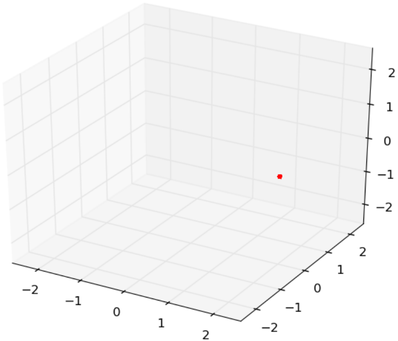
Direct integration with Gyoto ; $k = 2$ and $\omega = 0.7$.

Outer light ring



Direct integration with Gyoto ; $k = 2$ and $\omega = 0.7$.

Light point



Direct integration with Gyoto ; $k = 2$ and $\omega = 0.6387$.

Last words

- Boson stars are indeed very compact.
- Existence of two light rings (one stable).
- On each sequence existence of a single light point, where the photon is at rest.
- Not present in the BH case. Schwarzschild admit an unstable photon at rest trajectory, exactly on the horizon.
- Boson stars are good testbeds for strong relativistic effects.