On gravity duals for hot QCD

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based on works with
Dima Ageev, Irina Aref'eva and Eric Gourgoulhon

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Outline

The phase diagram of QCD

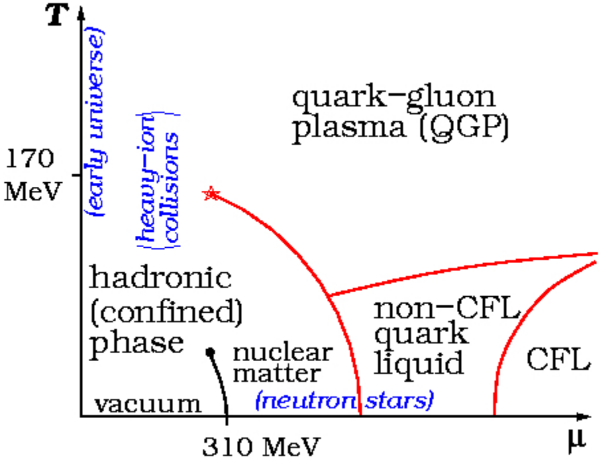

Figure: A sketch of the QCD phase diagram.

• QCD is a gauge theory of $SU(3)$ gauge group ($SU(3)$ Yang-Mills theory). Problems:

1. Wilson loops (the potential of interquark interaction)
2. β-function, RG-flow
3. Thermalization

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Methods:

1. Perturbative QCD (works only for α_s is small: high energies, short distances, i.e. asymptotic freedom)
2. Lattice QCD, numerical simulations, non-perturbative approach (the numerical sign problem for non-zero baryonic densities, no realtime calculations)
3. 1/N-expansion

   G.'t Hooft, A planar diagram theory for strong interactions *Nucl. Phys.* **B 72**, (1974) 461-473.

4. The gauge/gravity duality (e.g. the AdS/CFT correspondence), the contemporary form of large N expansion

Gravity helps strong interations

The AdS/CFT conjecture

The AdS/CFT correspondence Maldacena'98

The AdS/CFT conjecture claims exact equivalence between the theory in the bulk, which is a low energy approximation to $D = 10$ IIB string theory on $AdS_5 \times S^5$ the theory defined on the boundary, which is $\mathcal{N} = 4$ supersymmetric Yang-Mills with gauge group $SU(N_C)$ at large N_C.

- The strong coupling regime of one theory reflects the weak coupling regime of the other one, restore d-dim gauge theory thanks to the boundary $d + 1$-gravity dual

Gubser,Klebanov,Polyakov, Witten'98

$$Z_4[\phi_0(x)] = \int \mathcal{D} \exp\{iS_4 + \int_{x^4} \phi_0 \mathcal{O}\}, \quad S_4 = \int d^4x\mathcal{L}$$

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$$Z_5[\phi_0(x)] = \int_{\phi(x,\varepsilon)=\phi_0(x)} \mathcal{D}[\phi] e^{iS_5[\phi]}$$

Generating functional[4d sources $\phi_0(x)$] = Effective action[fields $\phi_0(x)$]

The AdS_{d+1}/CFT_d correspondence

$T \neq 0$, a deconfinement phase Witten'98

$$ds^2 = \frac{1}{z^2}\left[-f(z)dt^2 + (dx^i)^2 + \frac{dz^2}{f(z)}\right], \quad f(z) = 1 - \left(\frac{z}{z_h}\right)^4.$$

- The temperature of the the Yang-Mills theory is identified with the Hawking temperature of the black hole.

$\mu \neq 0$ RN-AdS black hole

$$ds^2 = \frac{1}{z^2}\left[-f(z)dt^2 + (dx^i)^2 + \frac{dz^2}{f(z)}\right], \quad f(z) = 1 - \frac{z^4}{z_h^4} - q^2 z_h^2 z^4 + q^2 z^6.$$

Does the conjecture work?

Does the conjecture work?

- checked (see, 1012.3982), but NOT prooved
- $\mathcal{N} = 4$ SYM with gauge group $SU(N_C)$ is a conformal theory (thanks to supersymmetry), QCD is not conformal
- however...

Lattice calculations show that QCD exhibits a quasi-conformal behavior at temperatures $T > 300\text{MeV}$ and the equation of state $\sim E = 3P$ (a traceless conformal energy-momentum tensor).

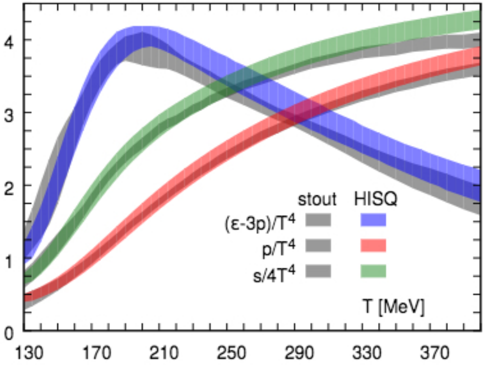

Figure: The comparison of the HISQ/tree and stout results for the trace anomaly, the pressure, and the entropy density

Pic. from Bazavov et al.'14

The quark-gluon plasma: quarks, antiquarks, gluons in deconfinement

$$\tau_{therm}(0.1 fm/c) < \tau_{hydro} < \tau_{hard}(10 fm/c) < \tau_f(20 fm/c)$$

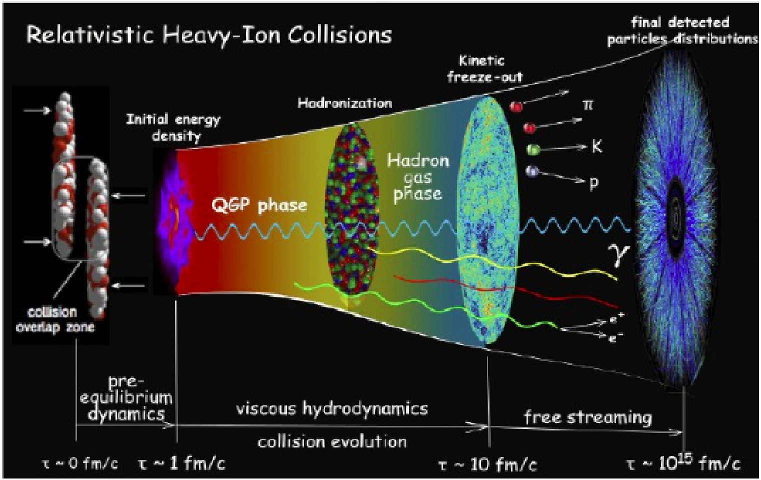

Figure: Picture from: P.Sorensen, C.Shen

The AdS/CFT correspondence for the QGP

- QGP does not behave like a weakly coupled gas of quarks and gluons, but a strongly coupled fluid.

$$\eta/s \approx \hbar/4\pi\kappa,$$

  the AdS/CFT calcs, Policastro, Son,Starinets'03
- confirmed at the RHIC'08

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- hadronization, freezing out: QGP is a particle factory
- early anisotropic stage: responsible for multiplicity of produced particles
- multiplicity (total number of particles produced in HIC)
- very fast thermalization

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Holographic models

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- Top-down approach: low-energy approximation of string theory (supergravity model) in asymptotically AdS backgrounds trying to reproduce features similar to QCD
  Examples:Sakai-Sugimoto model ($D4 - D8 - \bar{D}8$-branes), Mateos-Trancanelli model ($D3 - D7$-branes).

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  Examples:Sakai-Sugimoto model ($D4 - D8 - \bar{D}8$-branes), Mateos-Trancanelli model ($D3 - D7$-branes).
- **Bottom-up approach**: effective $5D$ gravitational theory with matter in
  - asymptotically AdS spacetimes
  - non-conformal backgrounds

  Examples: wall models (Karch et al., Erlich et al.), improved holographic QCD model (Kiritsis et al.)

Top-down approach :type IIB SUGRA, $D3 - D7$-branes

Anisotropic QGP , Mateos&Trancanelli'11

$$ds^2 = \frac{1}{u^2}\left(-\mathcal{F}\mathcal{B}dx_0^2 + dx_1^2 + dx_2^2 + \mathcal{H}dx_3^2 + \frac{du^2}{\mathcal{F}}\right) + \mathcal{Z}d\Omega_{S^5}^2 .$$

The functions $\mathcal{F}, \mathcal{B}, \mathcal{H}$ depend on the radial direction u and the anisotropy α. At high temperatures $\alpha \ll T$:

$\mathcal{F}(u) =$

$$1 - \frac{u^4}{u_h^4} + \frac{\alpha^2}{24u_h^2}\left[8u^2(u_h^2 - u^2) - 10u^4 \log 2 + (3u_h^4 + 7u^4)\log(1 + \frac{u^2}{u_h^2})\right]$$

$$\mathcal{B}(u) = 1 - \frac{\alpha^2}{24u_h^2}\left[\frac{10u^2}{u_h^2+u^2} + \log(1 + \frac{u^2}{u_h^2})\right], \ \mathcal{H}(u) = \left(1 + \frac{u^2}{u_h^2}\right)^{\frac{\alpha^2 u_h^2}{4}} .$$

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$D7$-probes in $D3$-background $Lif_{IR}/AdS_{5,UV} \times X_5 \Rightarrow$ deformed SYM.
$\alpha = 0 \Rightarrow$ isotropic $D3$-brane AdS/CFT: $AdS_5 \times S^5 \Rightarrow \mathcal{N} = 4$ SYM.
Jet quenching, drag force, potentials... see Giataganas et al.'12

Bottom-up: Breaking scale invariance

The AdS/CFT correspondence:

The Field Theory	The Gravitational Background
• the conformal group $SO(D,2)$	• the group of isometries
of a D-dimensional CFT	of AdS_{D+1}
$(t, x_i) \to (\lambda t, \lambda x_i)$, $i = 1, .., d-1$	$ds^2 = r^2\left(-dt^2 + d\vec{x}^2_{d-1}\right) + \frac{dr^2}{r^2}$

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Generalizations? ↗

Lifshitz scaling: $t \to \lambda^\nu t, \quad \vec{x} \to \lambda \vec{x}, \quad r \to \frac{1}{\lambda} r,$

where ν is the Lifshitz dynamical exponent

Lifshitz metric: $$ds^2 = -r^{2\nu} dt^2 + \frac{dr^2}{r^2} + r^2 d\vec{x}^2_{d-1}$$

Kachru, Liu, Millgan ’08

Holographic dictionary

We have to "mimic" the heavy ions collision

Models:

- shock waves collision in AdS
- infalling shell (Vaidya solutions)

Holographic dictionary

- $4d$ Multiplicity in HIC $=$ BH entropy in AdS_5 Gubster et al.'08

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- Wilson loops $=$ 2-dimensional minimal surfaces

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Lifshitz-like spacetimes

- A spatial extension of the Lifshitz scaling

$$(t, x, y, r) \to (\lambda^{\nu} t, \lambda^{\nu} x, \lambda y_1, \lambda y_2, \frac{r}{\lambda})$$

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$$ds^2 = r^{2\nu}\left(-dt^2 + dx^2\right) + r^2 dy_1^2 + r^2 dy_2^2 + \frac{dr^2}{r^2},$$

M. Taylor'08, Pal'09.

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The 5d Lifshitz-like metrics (boost-invariant)

$$\text{Type} - (\mathbf{1}, \mathbf{2}) \quad ds^2 = r^{2\nu}\left(-dt^2 + dx^2\right) + r^2\left(dy_1^2 + dy_2^2\right) + \frac{dr^2}{r^2}.$$

$$\text{Type} - (\mathbf{2}, \mathbf{1}) \quad ds^2 = r^{2\nu}\left(-dt^2 + dx_1^2 + dx_2^2\right) + r^2 dy^2 + \frac{dr^2}{r^2}.$$

Construction of a shock wave in Lifshitz-like spacetimes

The 5d Lifshitz-like metrics, $z = \dfrac{1}{r^\nu}$

$$\text{Type} - (\mathbf{1},\mathbf{2}) \quad ds^2 = L^2\left[\frac{(-dt^2+dx^2)}{z^2} + \frac{(dy_1^2+dy_2^2)}{z^{2/\nu}} + \frac{dz^2}{z^2}\right].$$

$$\text{Type} - (\mathbf{2},\mathbf{1}) \quad ds^2 = L^2\left(\frac{(-dt^2+dx_1^2+dx_2^2)}{z^2} + \frac{dy^2}{z^{2/\nu}} + \frac{dz^2}{z^2}\right).$$

A massless particle located at $u=0$ and moving with the speed of light in the v-direction:

$ds^2 = 2A(u,v)dudv + g(u,v)h_{ij}(x)dx^i dx^j$

$ds^2 = 2A(u,v)du(dv - f(x^i)\delta(u)du) + g(u,v)h_{ij}(x)dx^i dx^j,$

$v \to v + f(x).$

T. Dray & G. 't Hooft, '85; Hotta & Tanaka'93; Sfetsos'95.

The shock wave metric

$$ds^2 = \frac{\phi(y_1, y_2, z)\delta(u)}{z^2} du^2 - \frac{1}{z^2} du dv + \frac{1}{z^{2/\nu}} \left(dy_1^2 + dy_2^2\right) + \frac{dz^2}{z^2}, \quad (1)$$

$u = t - x$ and $v = t + x$ – light cone coordinates.

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$u = t - x$ and $v = t + x$ – light cone coordinates.

The equation for the profile

$$\left[\Box_{Lif_3} - \left(1 + \frac{2}{\nu}\right)\right] \frac{\phi(y_1, y_2, z)}{z} = -2z t_{uu}, \quad T_{uu} = t_{uu}\delta(u), \quad (2)$$

$$\Box_{Lif_3} = \frac{1}{\nu}\left(z^2 \nu \frac{\partial^2}{\partial z^2} + \nu z \frac{\partial}{\partial z} - 2z \frac{\partial}{\partial z} + z^{2/\nu} \nu \frac{\partial^2}{\partial y_1^2} + \nu z^{2/\nu} \frac{\partial^2}{\partial y_2^2}\right).$$

$ds^2_{Lif_3} = \frac{dy_1^2 + dy_2^2}{z^{2/\nu}} + \frac{dz^2}{z^2}.$

Domain-walls in Lifshitz-like spacetimes

$$\phi(y_1, y_2, z) = \phi(z),$$ Lin & Shuryak'09

The equation for the profile

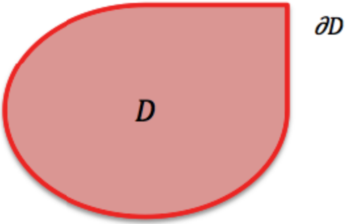
$$\begin{aligned}
\frac{\partial^2\phi(z)}{\partial z^2} &- \left(1+\frac{2}{\nu}\right)\frac{1}{z}\frac{\partial\phi(z)}{\partial z} = -16\pi G_5 E\left(\frac{z}{L}\right)^{1+2/\nu}\delta(z-z_*).\\
\phi &= \phi_a\Theta(z_*-z)+\phi_b\Theta(z-z_*),\\
\phi_a(z) &= C_0 z_a z_b\left(\frac{z_*^{2(\nu+1)/\nu}}{z_b^{2(\nu+1)/\nu}}-1\right)\left(\frac{z^{2(\nu+1)/\nu}}{z_a^{2(\nu+1)/\nu}}-1\right),\\
\phi_b(z) &= C_0 z_a z_b\left(\frac{z_*^{2(\nu+1)/\nu}}{z_a^{2(\nu+1)/\nu}}-1\right)\left(\frac{z^{2(\nu+1)/\nu}}{z_b^{2(\nu+1)/\nu}}-1\right),\\
C_0 &= -\frac{8\nu\pi G_5 E z_a^{1+2/\nu} z_b^{1+2/\nu}}{(\nu+1)L^{3+\frac{2}{\nu}}\left(z_b^{2(\nu+1)/\nu}-z_a^{2(\nu+1)/\nu}\right)}.
\end{aligned}$$

Trapped surfaces

Theorem:

$\exists$ TS for two shock waves = $\exists$ solution to the following Dirichlet problem:

- $\Psi_{1,2} > 0$ for $X \in D$ and $\Psi_{1,2} = 0$ for $X \in \partial D$
- $\nabla^2 \Psi_{1,2} = \delta(X - X_{(1,2)}), \quad X \in D$
- $\nabla \Psi_1 \nabla \Psi_2 = 4, \quad X \in \partial D$



Eardley & Giddings'03;
Kang & Nastase'05

Wall-on-wall collisions

$$ds^2 = -\frac{1}{z^2}dudv + \frac{\phi_1(z)}{z^2}\delta(u)du^2 + \frac{\phi_2(z)}{z^2}\delta(v)dv^2 + \frac{dy^2}{z^2} + \frac{dw^2}{z^{4/3}} + \frac{dz^2}{z^2}.$$

The trapped surface for wall-on-wall shock wave collision is $z_a < z_* < z_b$

.

$$(\partial_z\phi)|_{z=z_a} = 2, \quad (\partial_z\phi)|_{z=z_b} = -2$$

$$\Updownarrow$$

$$\frac{8\pi G_5 E z_a^{8/3}\left(1-\frac{z_b^{11/3}}{z_*^{11/3}}\right)}{\tilde{R}^{14/3}\left(\frac{z_b^{11/3}}{z_*^{11/3}}-\frac{z_a^{11/3}}{z_*^{11/3}}\right)} = -1, \quad \frac{8\pi G_5 E z_b^{8/3}\left(1-\frac{z_a^{11/3}}{z_*^{11/3}}\right)}{\tilde{R}^{14/3}\left(\frac{z_b^{11/3}}{z_*^{11/3}}-\frac{z_a^{11/3}}{z_*^{11/3}}\right)} = 1,$$

$$z_a = \left(\frac{z_b^{8/3}}{-1+z_b^{8/3}C}\right)^{3/8}, \; z_* = \left(\frac{z_a^{8/3}z_b^{8/3}(z_a-z_b)}{z_a^{8/3}-z_b^{8/3}}\right)^{3/8}.$$

The area of the trapped surface

$$S = \frac{1}{2G_5} \int_C \sqrt{\det|g_{Lif_3}|} dz dy_1 dy_2,$$

where $\det g_{Lif_3}$ is the metric determinant for

$$ds^2_{Lif_3} = z^{-2/\nu}\left(dy_1^2 + dy_2^2\right) + \frac{dz^2}{z^2}.$$

The relative area s of the trapped surface defined by

$$\mathbf{s} = \frac{S_{\text{trap}}}{\int dy_1 dy_2} = \frac{\nu}{4G_5}\left(\frac{1}{(z_a)^{2/\nu}} - \frac{1}{(z_b)^{2/\nu}}\right).$$

$$s(C, z_b) = \left(\frac{C}{2}\right)^{\frac{2}{2+\nu}} - \left(\frac{1}{z_b}\right)^{\frac{2}{\nu}} - \frac{2}{(\nu+2)}\left(\frac{2}{C}\right)^{\frac{2}{2+\nu}}\left(\frac{1}{z_b}\right)^{\frac{2+\nu}{\nu}} + \quad \ldots \quad .$$

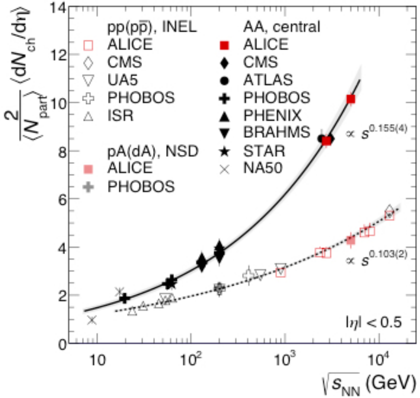
The maximum value at infinite z_b

$$\mathbf{s}|_{z_b\to\infty} = \frac{\nu}{4G_5}(8\pi G_5)^{2/(\nu+2)} E^{2/(\nu+2)}$$

Experiment:
$S_{data} = \mathbf{s}_{NN}^{0.155}$
ALICE collaboration'15

AdS with ghosts:
$S_{data} = \mathbf{s}_{NN}^{0.12}$
Kiritis & Taliotis'11

AdS+ ghosts:
$S_{data} = \mathbf{s}_{NN}^{0.16}$
Aref'eva et al.'14



ALICE collaboration'15

Deformed AdS
$S_{data} = \mathbf{s}_{NN}^{0.16}$
Aref'eva & A.G.'14

Black branes in Lifshiz-like spacetimes

$$S = \frac{1}{16\pi G_5}\int d^5x\sqrt{|g|}\left(R[g] + \Lambda - \frac{1}{2}\partial_M\phi\partial^M\phi - \frac{1}{4}e^{\lambda\phi}F_{(2)}^2\right),$$

Λ is negative cosmological constant.

The Einstein equations

$$R_{mn} = -\frac{\Lambda}{3}g_{mn} + \frac{1}{2}(\partial_m\phi)(\partial_n\phi) + \frac{1}{4}e^{\lambda\phi}\left(2F_{mp}F_n^p\right) - \frac{1}{12}e^{\lambda\phi}F^2 g_{mn}.$$

The scalar field equation

$$\Box\phi = \frac{1}{4}\lambda e^{\lambda\phi}F^2, \quad \text{with} \quad \Box\phi = \frac{1}{\sqrt{|g|}}\partial_m(g^{mn}\sqrt{|g|}\partial_n\phi).$$

The gauge field

$$D_m\left(e^{\lambda\phi}F^{mn}\right) = 0.$$

The Lifshitz-like black brane

$$ds^2 = e^{2\nu r}\left(-f(r)dt^2 + dx^2\right) + e^{2r}\left(dy_1^2 + dy_2^2\right) + \frac{dr^2}{f(r)},$$

where $f(r) = 1 - me^{-(2\nu+2)r}$. Aref'eva,AG, Gourgoulhon'16

$$F_{(2)} = \frac{1}{2}q\,dy_1 \wedge dy_2, \quad \phi = \phi(r), \quad e^{\lambda\phi} = \mu e^{4r}.$$

The Hawking temperature of the black brane:

$$T = \frac{1}{\pi}\frac{(\nu+1)}{2\nu}\,m^{\frac{\nu}{2\nu+2}}.$$

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$$F_{(2)} = \frac{1}{2} q dy_1 \wedge dy_2, \quad \phi = \phi(r), \quad e^{\lambda\phi} = \mu e^{4r}.$$

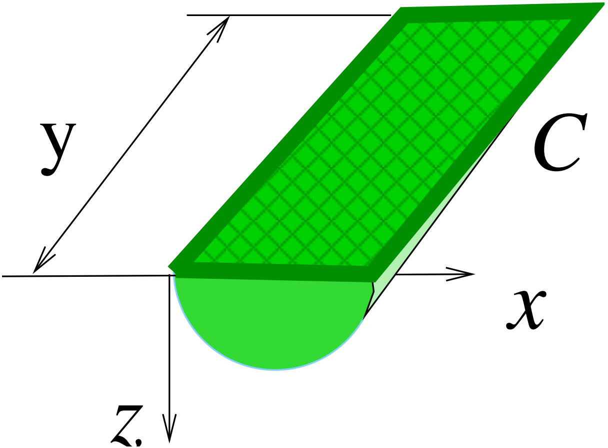
The Hawking temperature of the black brane:

$$T = \frac{1}{\pi}\frac{(\nu+1)}{2\nu}\, m^{\frac{\nu}{2\nu+2}}.$$

$$ds^2 = \frac{(-f(z)dt^2 + dx^2)}{z^2} + \frac{(dy_1^2 + dy_2^2)}{z^{2/\nu}} + \frac{dz^2}{z^2 f(z)},$$

$$f(z) = 1 - mz^{2+2/\nu}, \quad z = \frac{1}{r^\nu}.$$

Holographic spatial Wilson loops



Holographic Wilson Loops

- The expectation value of WL in the fundamental representation calculated on the gravity sided Maldacena'98, Rey et al.'98, Sonnenschein et al.98

$$W[C] = \langle \mathrm{Tr}_F \, e^{i \oint_C dx_\mu A_\mu} \rangle = e^{-S_{string}[C]}.$$

The Nambu-Goto action is

$$S_{string} = \frac{1}{2\pi\alpha'} \int d\sigma^1 d\sigma^2 \sqrt{-\det(h_{\alpha\beta})}, \tag{3}$$

with the induced metric of the world-sheet $h_{\alpha\beta}$ given by

$$h_{\alpha\beta} = g_{MN} \partial_\alpha X^M \partial_\beta X^N, \quad \alpha, \beta = 1, 2, \tag{4}$$

g_{MN} is the background metric, $X^M = X^M(\sigma^1, \sigma^2)$ specify the string, σ^1, σ^2 parametrize the worldsheet.

- The potential of the interquark interaction

$$W(T, X) = \langle \mathrm{Tr} \, e^{i \oint_{T\times X} dx_\mu A_\mu} \rangle \sim e^{-V(X)T}.$$

The infalling shell background

The ingoing Eddington-Finkelstein coordinates

$$dv = dt + \frac{dz}{f(z)}.$$

The Vaidya solution in Lifshitz background

$$\begin{aligned} ds^2 &= -z^{-2}f(z)dv^2 - 2z^{-2}dvdz + z^{-2}dx^2 + z^{-2/\nu}(dy_1^2 + dy_2^2), \\ f &= 1 - m(v)z^{2+2/\nu}, v < 0 - \text{inside the shell}, v > 0 - \text{outside}, \\ f(v,z) &= 1 - \frac{M}{2}\left(1 + \tanh\frac{v}{\alpha}\right) z^{2+\frac{2}{\nu}} \end{aligned}$$

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- The Vaidya solution interpolates between the black hole (outside the shell) and the Lifshitz-like vacuum (inside the shell).

Balasubramanian et. al.'11

WL in time-dependent backgrounds.Case 1

$$v = v(x), \quad z = z(x), \quad f = f(v,z).$$

$$S_{x,y_{1(\infty)}} = \frac{L_y}{2\pi\alpha'} \int \frac{dx}{z^{1+1/\nu}} \sqrt{1 - f(z,v) v'^{\,2} - v' z'}, \qquad ' \equiv \frac{d}{dx}.$$

The corresponding equations of motion are

$$\begin{aligned} v'' &= \frac{1}{2}\frac{\partial f}{\partial z} v'^2 + \frac{(\nu+1)}{\nu z}(1 - f v'^2 - 2v'z'), \\ z'' &= -\frac{\nu+1}{\nu}\frac{f}{z} + \frac{\nu+1}{\nu}\frac{f^2 v'^2}{z} - \frac{1}{2}\frac{\partial f}{\partial v} v'^2 - \frac{1}{2} f v'^2 \frac{\partial f}{\partial z} - v' z' \frac{\partial f}{\partial z}, \\ &+ 2\frac{(\nu+1)}{\nu z} f v' z'. \end{aligned}$$

The boundary conditions $z(\pm\ell) = 0$, $v(\pm\ell) = t$. The initial conditions $z(0) = z_*$, $v(0) = v_*$, $z'(0) = 0, v'(0) = 0$. The pseudopotential is

$$\mathcal{V}_{x,y_{1(\infty)}} = \frac{S_{x,y_{1(\infty)},ren}}{L_{y_1}}$$

$$\delta\mathcal{V}_1(x,t) = \mathcal{V}_{x,y_{1(\infty)}}(x,t) - \mathcal{V}_{x,y_{1(\infty)}}(x,t_f).$$

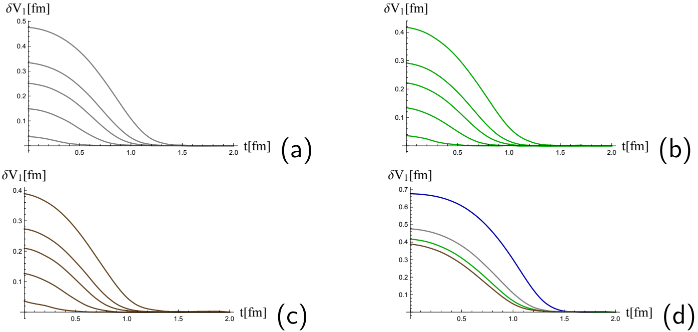

Figure: The time dependence of $-\delta\mathcal{V}_1(x,t)$, for different values of the length ℓ, $\nu = 2, 3, 4$ ((a),(b),(c), respectively). Different curves correspond to $\ell = 0.7, 1.2, 1.5, 1.7, 2$ (from down to top, respectively). In (d) we have shown $-\delta\mathcal{V}_1(x,t)$ as a function of t at $\ell = 2$ for $\nu = 1, 2, 3, 4$ (from top to down).

WL in time-dependent backgrounds.Case 2

$$v = v(y_1), \quad z = z(y_1), \quad f = f(v,z)$$

$$S_{y_1, x_{(\infty)}} = \frac{L_x}{2\pi\alpha'} \int dy_1 \frac{1}{z^2} \sqrt{\left(\frac{1}{z^{2/\nu - 2}} - f(z,v)(v')^2 - 2v'z' \right)}, \quad ' \equiv \frac{d}{dy_1}.$$

The corresponding equations of motion are

$$\begin{aligned} v'' &= \frac{1}{2}\frac{\partial f}{\partial z} v'^2 + \frac{\nu+1}{\nu z}\left(z^{2-2/\nu} - \frac{2\nu}{(1+\nu)} f v'^2 - 2v'z' \right), \\ z'' &= -\frac{\nu+1}{\nu} f z^{1-2/\nu} + \frac{2(\nu-1)z'^2}{\nu} + \frac{2}{\nu}\frac{f^2 v'^2}{z} - \frac{1}{2\nu}\frac{\partial f}{\partial v} v'^2 - \frac{1}{2\nu} f \frac{\partial f}{\partial z} v'^2 \\ &- z'v'\frac{\partial f}{\partial z} + \frac{4}{z} f z' v'. \end{aligned}$$

The boundary conditions $z(\pm\ell) = 0$, $v(\pm\ell) = t$. The initial conditions $z(0) = z_*$, $v(0) = v_*$, $z'(0) = 0, v'(0) = 0$. The pseudopotential is

$$\mathcal{V}_{y_1, x_{(\infty)}} = \frac{S_{y_1, x_{(\infty)}, ren}}{L_{y_1}}.$$

WL in time-dependent backgrounds.Case 2

$$\delta\mathcal{V}_{y_1,\,x_{(\infty)}}(x,t)=\mathcal{V}_{y_1,\,x_{(\infty)}}(x,t)-\mathcal{V}_{y_1,\,x_{(\infty)}}(x,t_f).$$

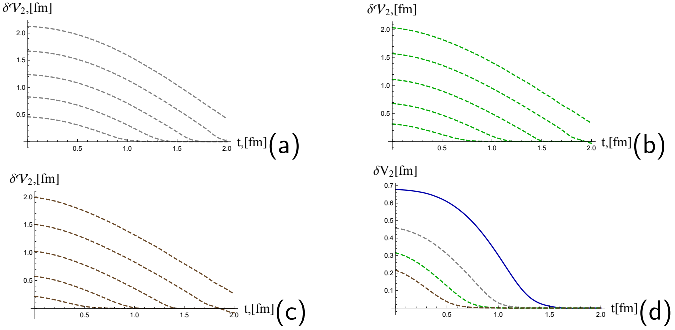

Figure: The time dependence of $-\delta\mathcal{V}_{y_1,\,x_{(\infty)}}(x,t)$ for different values of the length ℓ, $\nu=2,3,4$ ((a),(b),(c), respectively). Different curves correspond to $\ell=2,2.5,3,3.5,4$ (from down to top, respectively). In (d) $-\delta\mathcal{V}_2(x,t)$ as a function of t at $\ell=2$ for $\nu=1,2,3,4$ (from top to down, respectively).

WL in time-dependent backgrounds.Case 3

$$v=v(y_1),\quad z=z(y_1),\quad f=f(v,z).$$

$$S_{y_1,\,y_{2,(\infty)}}=\frac{L_{y_2}}{2\pi\alpha'}\int dy_1\frac{1}{z^{1+1/\nu}}\sqrt{\left(\frac{1}{z^{2/\nu-2}}-f(v')^2-2v'z'\right)}.$$

The corresponding equations of motion are

$$\begin{aligned}
v'' &= \frac{1}{2}\frac{\partial f}{\partial z}v'^2+\frac{2}{z\nu}\left(z^{2-2/\nu}-\frac{\nu+1}{2}fv'^2-2v'z'\right),\\
z'' &= -\frac{2}{\nu}fz^{1-2/\nu}+2\frac{\nu-1}{\nu}\frac{z'^2}{z}+\frac{\nu+1}{\nu z}f^2v'^2-\frac{1}{2}\frac{\partial f}{\partial v}v'^2-\frac{1}{2}f\frac{\partial f}{\partial z}v'^2\\
&- z'v'\frac{\partial f}{\partial z}+\frac{2(\nu+1)}{\nu z}fv'z'. \qquad (5)
\end{aligned}$$

The boundary conditions $z(\pm\ell)=0$, $v(\pm\ell)=t$. The initial conditions $z(0)=z_*$, $v(0)=v_*$, $z'(0)=0, v'(0)=0$. The pseudopotential is

$$\mathcal{V}_{y_1,\,y_{2,(\infty)}}(t,\ell)=\frac{S_{y_1,\,y_{2,(\infty)},ren}}{L_{y_2}}.$$

WL in time-dependent backgrounds.Case 3

$$\delta\mathcal{V}_{y_1,\,y_{2,(\infty)}}(x,t) = \mathcal{V}_{y_1,\,y_{2,(\infty)}}(x,t) - \mathcal{V}_{y_1,\,y_{2,(\infty)}}(x,t_f).$$

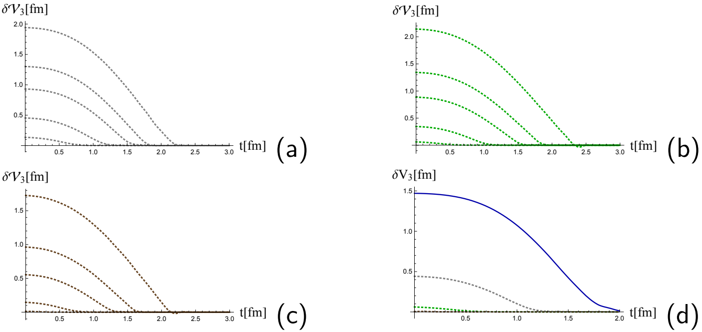

Figure: $-\delta\mathcal{V}_{y_1,\,y_{2,(\infty)}}(x,t)$ on t for different ℓ, $\nu = 2,3,4$ ((a),(b),(c)). (a): $l = 2.2, 3, 3.85, 4.4, 5.2$ from top to down; (b): $l = 3, 4.1, 5.2, 6, 7.1$ from top to down; (c): $l = 3.4, 4.6, 5.9, 6.8, 8$ from top to down. In (d): $-\delta\mathcal{V}_3(x,t)$ as a function of t at $\ell = 3$ for $\nu = 1,2,3,4$ (from top to down, respectively).

$\nu = 4$

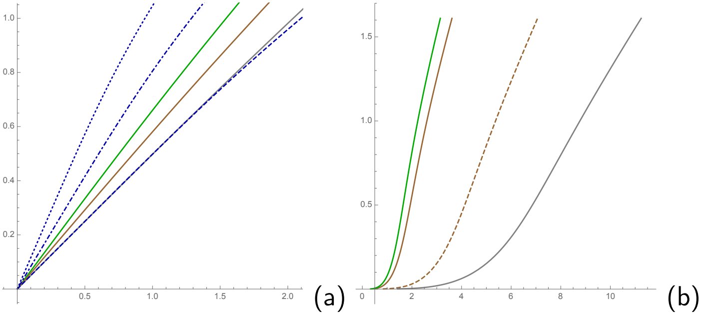

Figure: The thermalization times of the two-point correlators, holographic entanglement entropy and WL. (a)The solid lines (from left to right) correspond to the entropy(green), the Wilson loop (brown) and the two-point correlator (gray) with the dependences on the longitudinal direction x , while the dashed, dash-dotted and dotted lines represent the behaviour of the two-point correlator, Wilson loop and entropy in the isotropic spacetime, respectively. (b)The solid curves correspond to the entropy(green), the Wilson loop (brown) on the xy_1-plane and the two-point correlator (gray) with the dependences on the trasversal direction y_1.

Outline

Summary and Outloook

Done

1. Black brane and shell solutions with Lifshitz-like asymptotics
2. Wilson loops in the Lifshitz-like backgrounds
3. Pseudopotentials and spatial string tensions

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Open questions

1. Time-like Willson loops, potentials, quarkonium spectrum that CAN FIT experemental data for multiplicity
2. Generalization for non-zero chemical potential (non-zero baryon density)
3. Holographic RG-flow between two fixed points which correspond to the gravity solutions with different asymptotics
4. Any supergravity embeddings?

Je vous remercie de votre attention!