Test of special relativity using a fiber network of optical clocks

P. DELVA

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The Robertson–Mansouri–Sexl (RMS) framework

- Kinematical framework which assumes the existence of a preferred frame Σ where light propagates rectilinearly and isotropically in free space with constant speed c
  [Robertson, 1949, Mansouri and Sexl, 1977a, Mansouri and Sexl, 1977b, Mansouri and Sexl, 1977c]
- Ordinary Lorentz transformations from Σ to the observer frame S with relative velocity $\vec{w}$ are generalized to allow for violations of SR:

$$\begin{aligned} T &= a^{-1}(t - c^{-1}\vec{\epsilon}\cdot\vec{x}) \\ \vec{X} &= d^{-1}\vec{x} - (d^{-1} - b^{-1})(\vec{w}\cdot\vec{x})\vec{w}/w^2 + \vec{w}T \ , \end{aligned}$$

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- In the low-velocity limit:

$$a(\vec{w}) = 1 + c^{-2}(\alpha - 1/2)w^2 + \mathcal{O}(c^{-4}w^4) \qquad (1)$$

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- Three classical LI tests: Michelson–Morley, Kennedy–Thorndike, and Ives–Stillwell experiments [Robertson, 1949]
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JPL time of flight	1.8×10^{-4}
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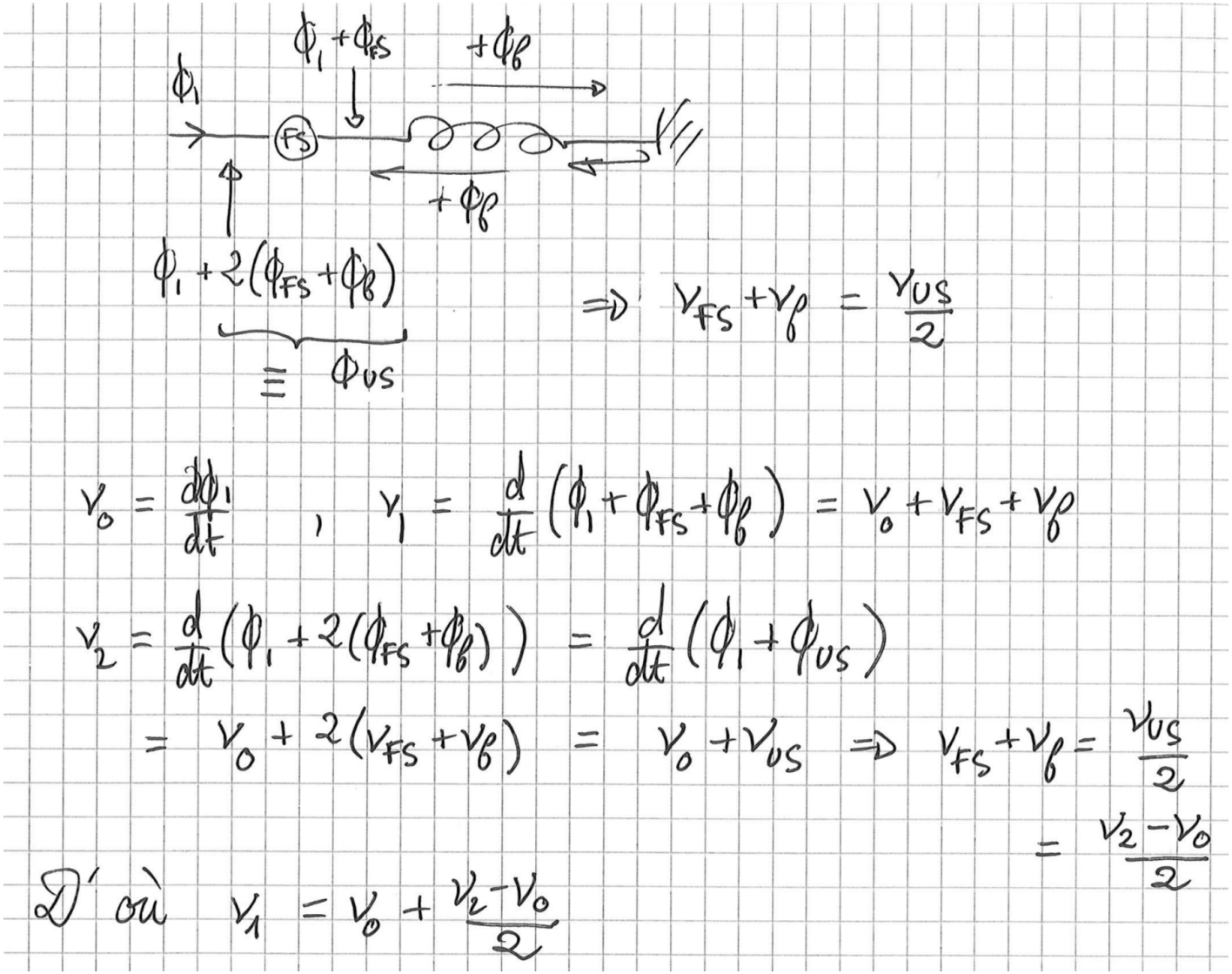
Simplified model of compensated fibre frequency transfer



Simplified model of compensated fibre frequency transfer

$$\frac{\nu_1}{\nu_0} = 1 + \frac{\nu_2 - \nu_0}{2\nu_0} + \Delta_{\mathrm{GR}} + \Delta_{\mathrm{RMS}}$$

- Δ_{GR} contains the relativistic redshift due to the static part of the gravity potential as well as temporal variations [Voigt et al., 2016].
- The RMS signal is:

$$\Delta_{\mathrm{RMS}} = \alpha c^{-2} \left[2\vec{w} \cdot (\vec{v}_A - \vec{v}_B) + (v_A^2 - v_B^2) \right] + \mathcal{O}(c^{-3})$$

where $\vec{v}_A$ and $\vec{v}_B$ are velocities of clocks A and B in GCRS, and $\vec{w}$ is velocity of Earth w.r.t. CMB

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Static and varying geopotential correction

Correcting for the gravitational redshift due to the static potential

- ITOC project (2013–2016, EMRP): new European geoid EGG2015, GNSS/geoid levelling (H. Denker et al., LUH)

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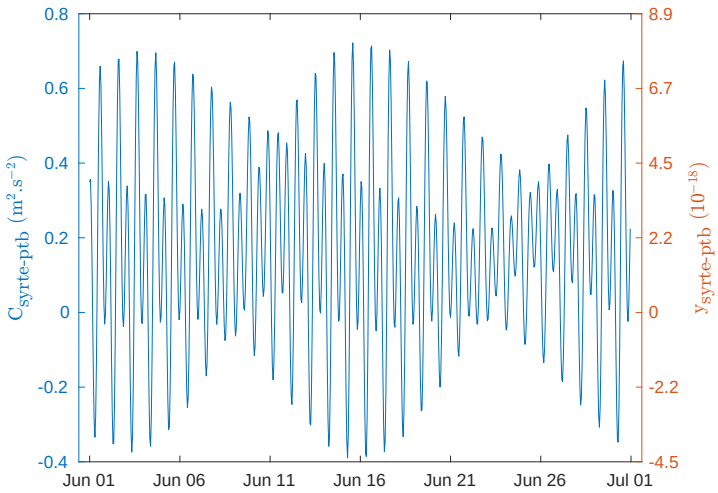

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C. Voigt et al., 2016. Metrologia 53, 1365.

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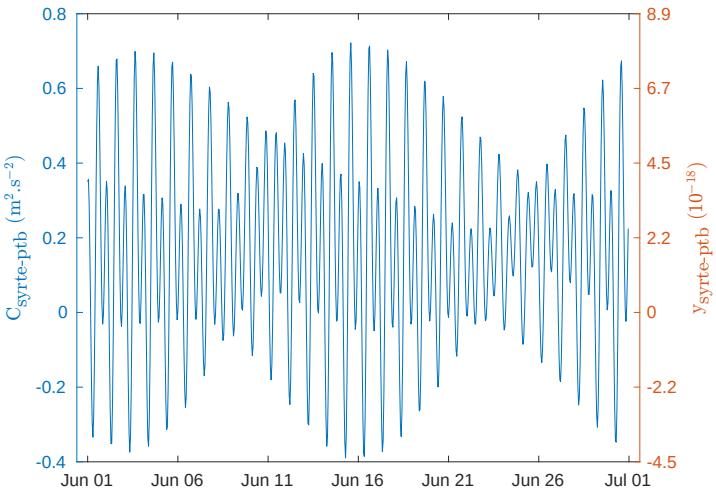

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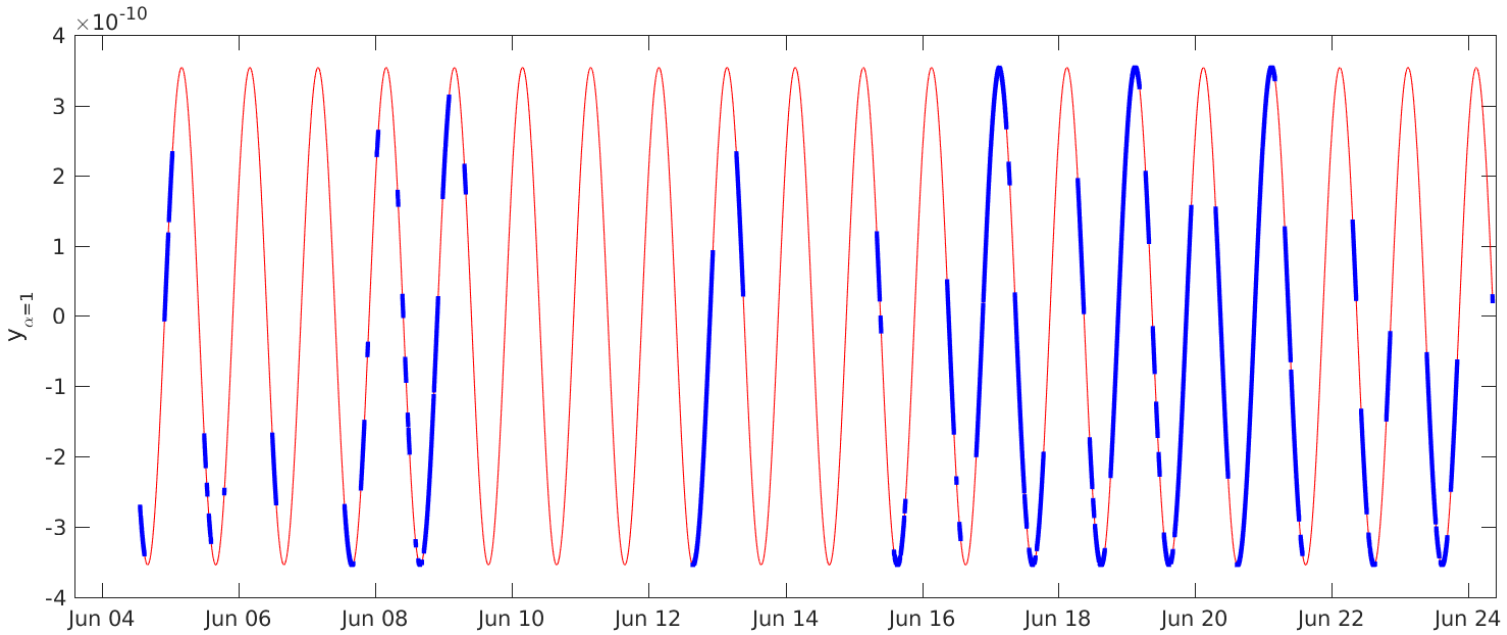

Clock comparison between PTB and SYRTE

$$\Delta_{\mathrm{RMS}} = 2\alpha c^{-2}\vec{w}\cdot(\vec{v}_A - \vec{v}_B) + \mathcal{O}(c^{-2})$$

- Velocities of the clocks in GCRS are obtained with the SOFA routines
- Δ_{RMS} varies daily for clocks located at different longitudes (direction) and/or different latitudes (norm)

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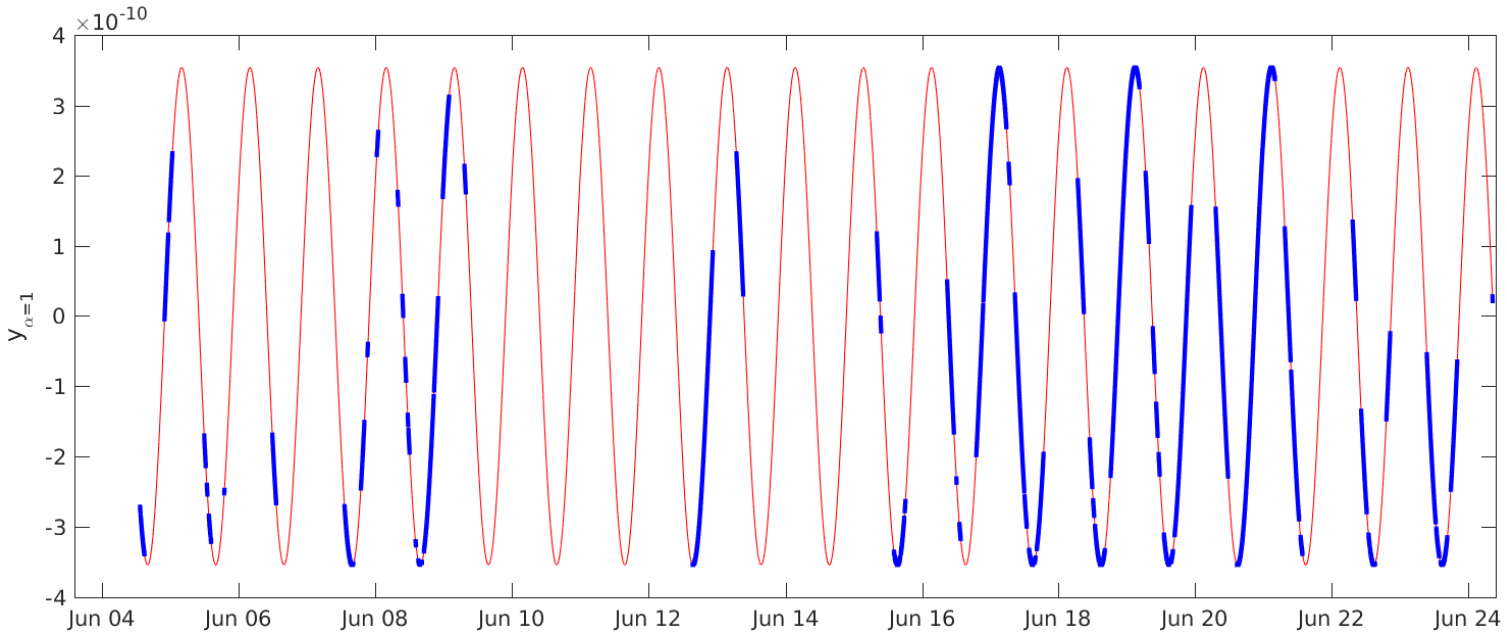

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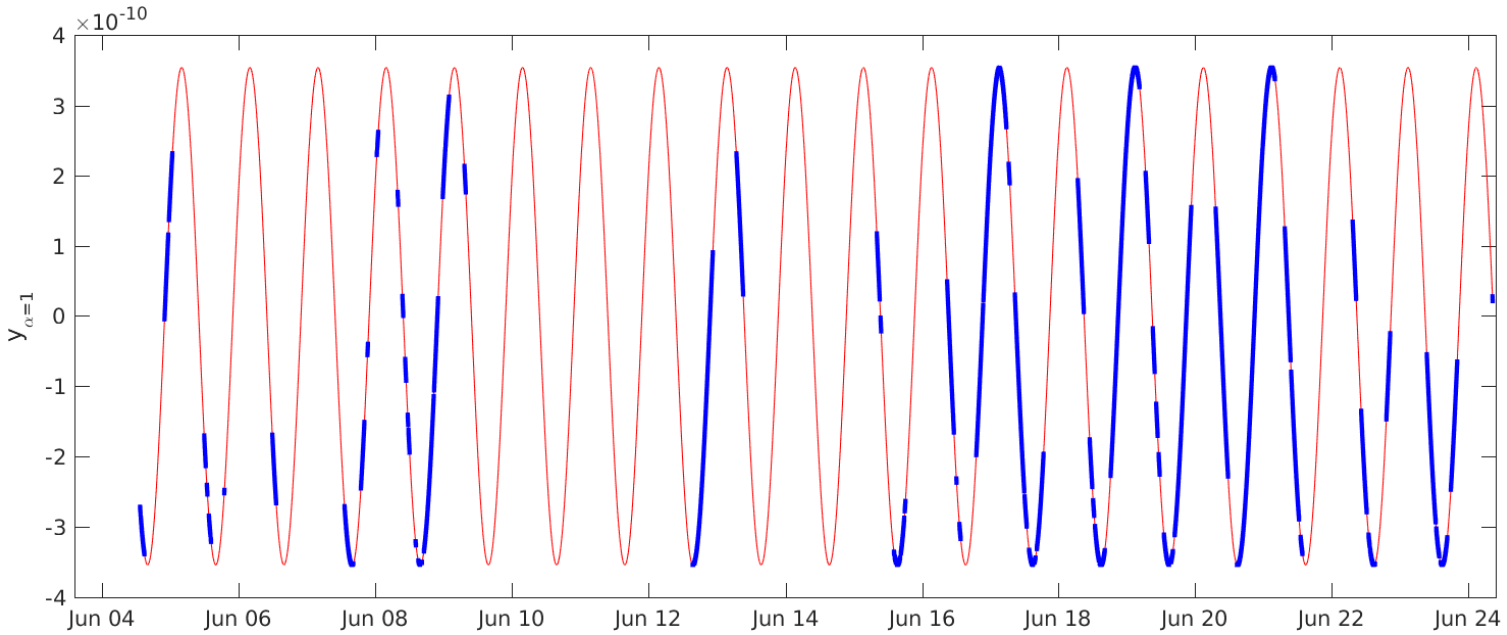

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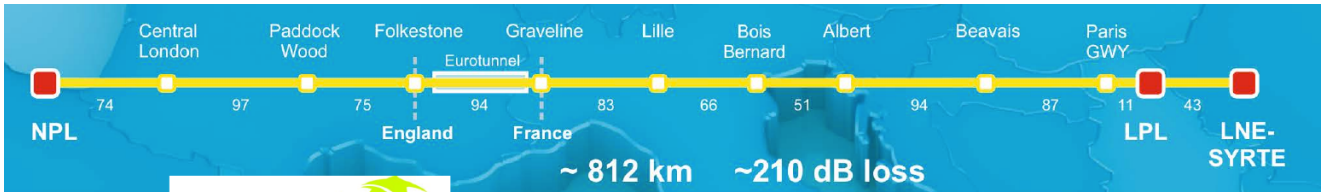
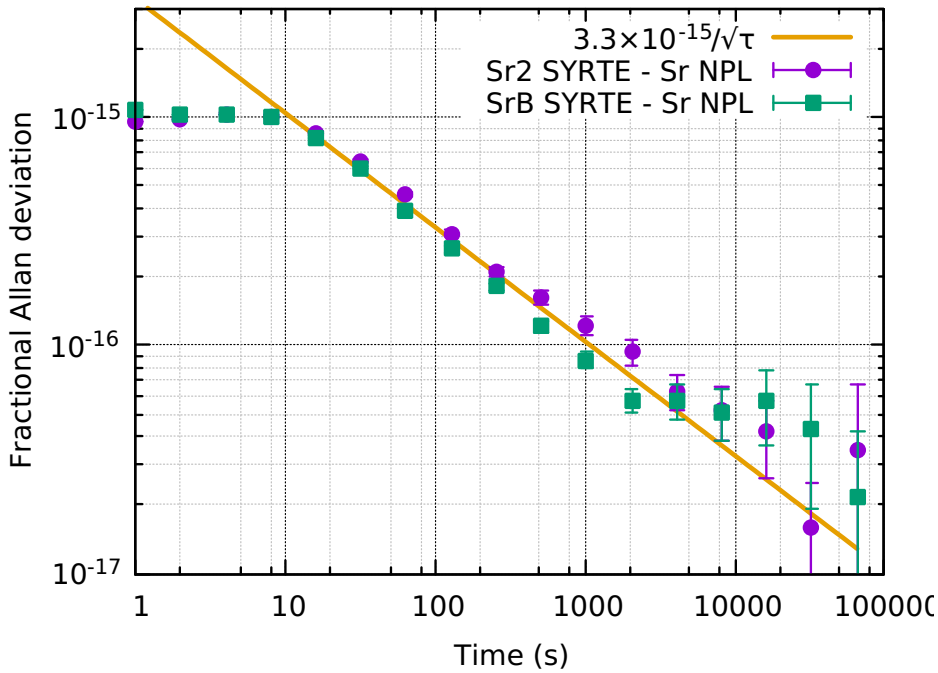
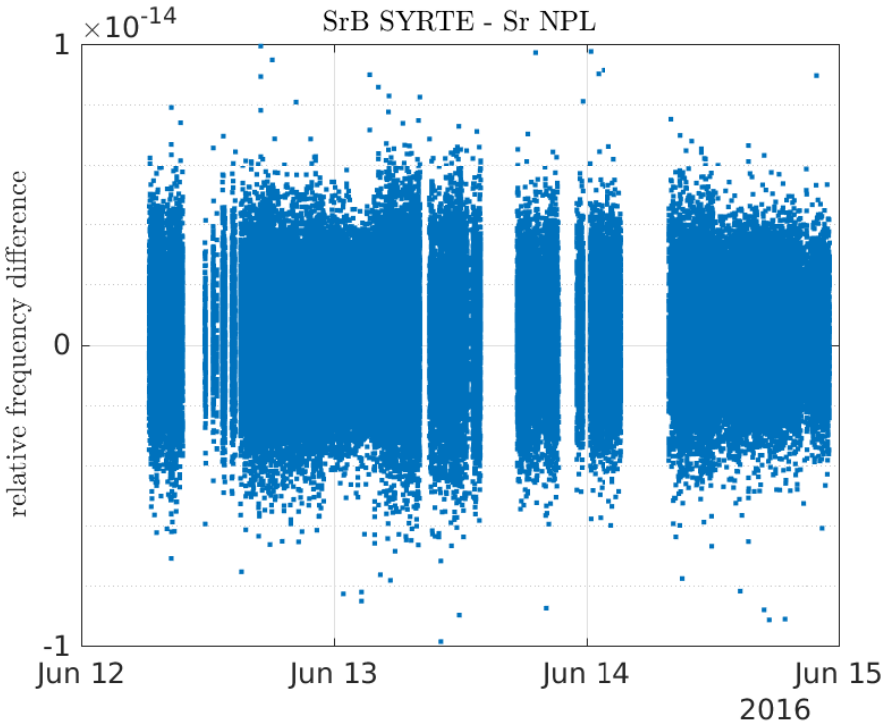
The Sr clock NPL-SYRTE comparison

Fiber link SYRTE – LPL – NPL first operated in June 2016



Sr vs. Sr

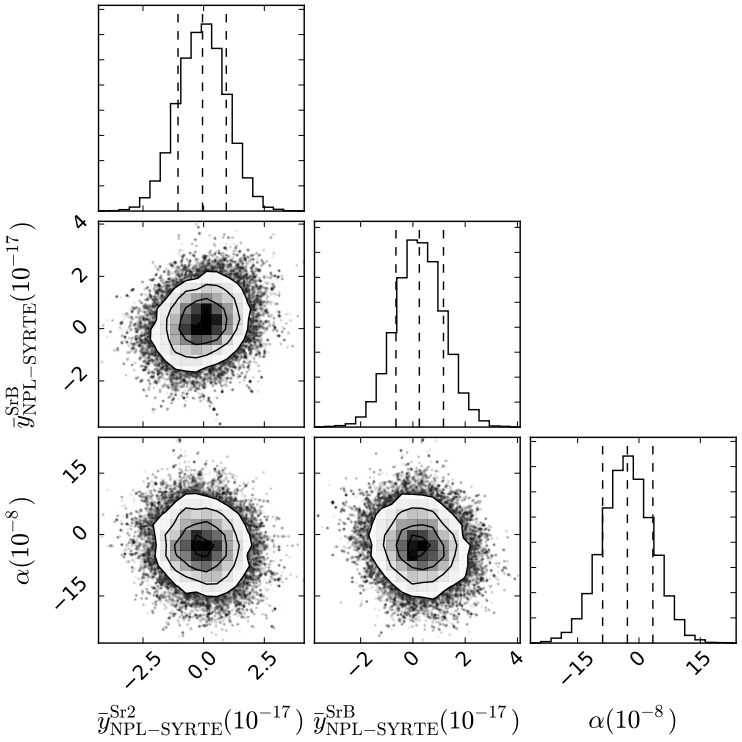




42 h of data over a few days

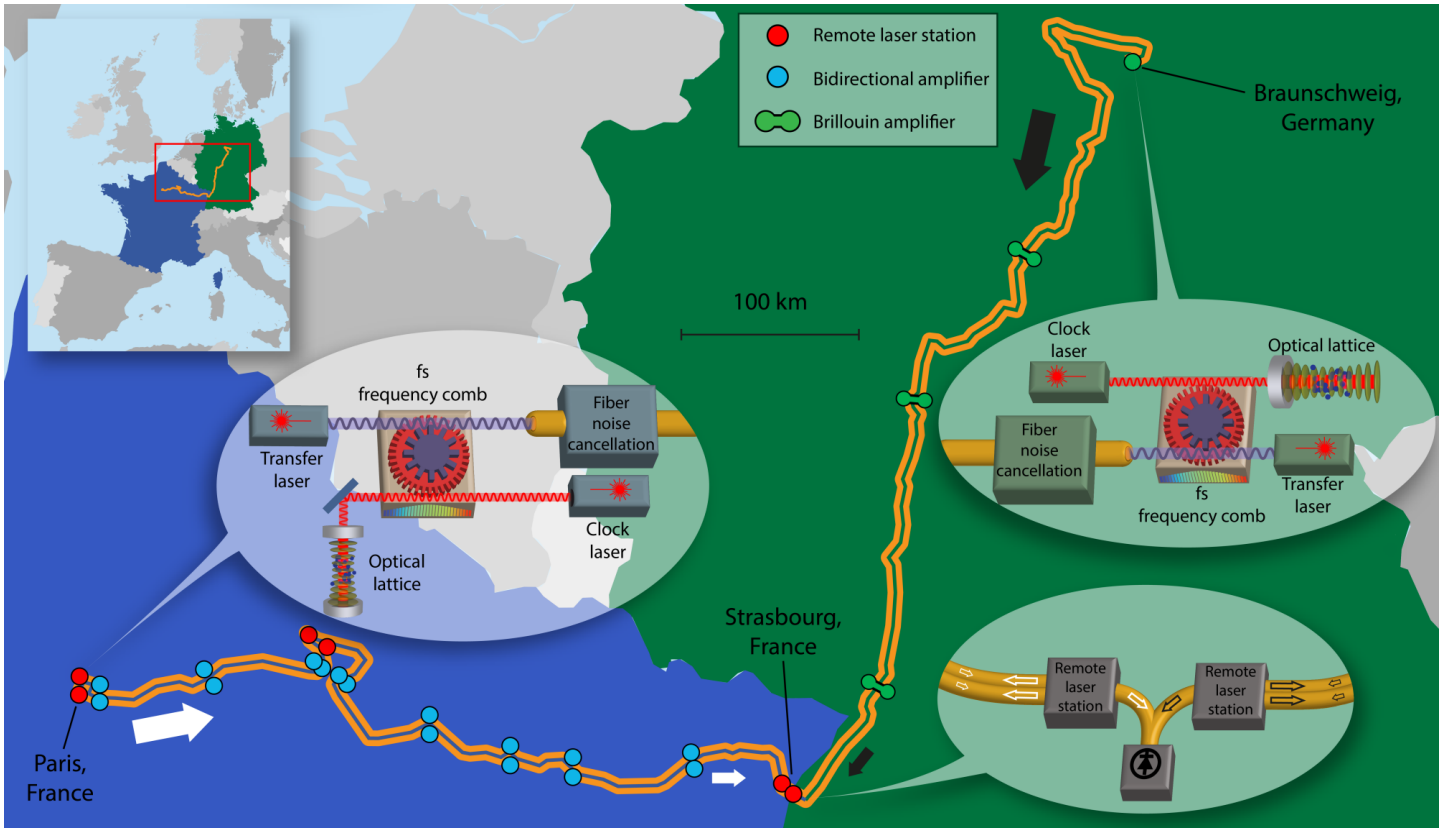
The Sr clocks NPL-SYRTE comparison

	Sr2 SYRTE - Sr NPL	SrB SYRTE - Sr NPL	Combined
$\alpha(10^{-8})$	$+3.81 \pm 8.41$	-5.87 ± 7.78	-2.83 ± 6.19



- Two different fitting methods: linear least-square fitting (LSQ) and affine invariant Markov Chain Monte Carlo ensemble sampler (MCMC)
- Temporal correlation due to flicker floor of the free running laser (up to 10 s)
- Correlation between the two data sets: Sr NPL clock is common for some points

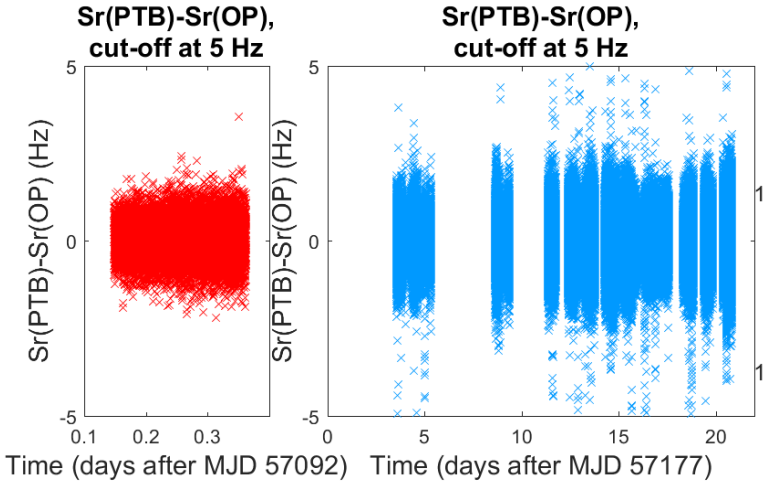
The Sr clocks PTB-SYRTE comparison



PTB, LPL and SYRTE established a 1415 km long optical fibre link and performed in 2015 the first direct comparison of optical clocks at continental scale

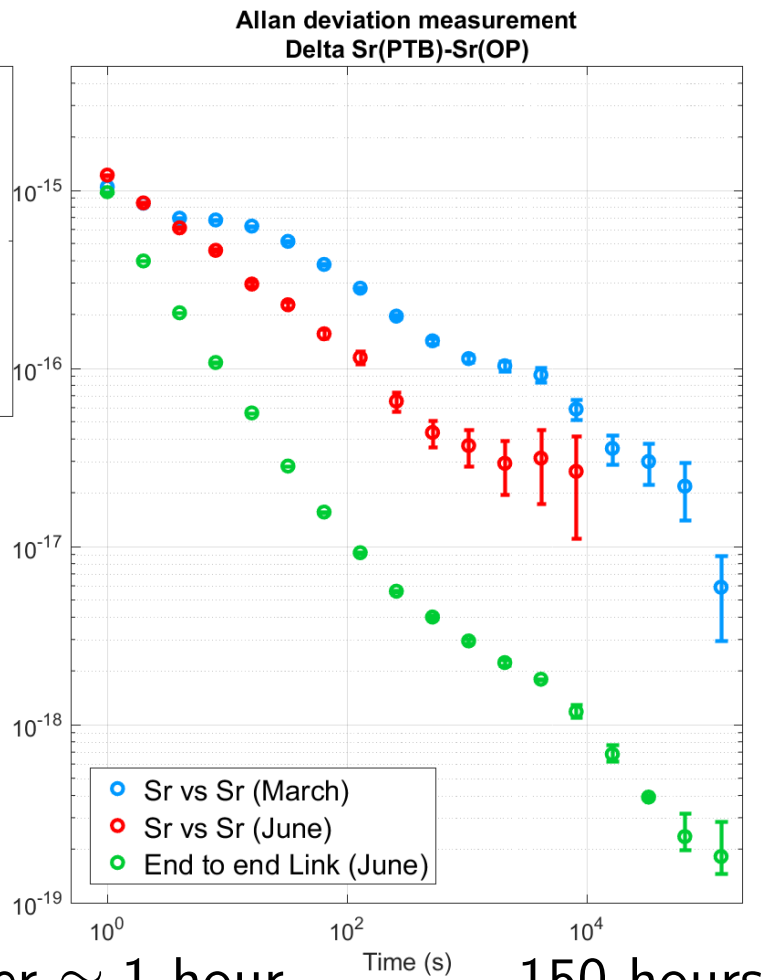
The Sr clocks PTB-SYRTE comparison

2 Measurement campaigns



Total clocks	650.3	4.1	496.3	1.9

Ratio Sr_{PTB}/Sr_{SYRTE}	Campaign I Unc. (10^{-17})	Campaign II Unc. (10^{-17})
Systematics Sr_{SYRTE}	4.1	4.1
Systematics Sr_{PTB}	2.1	1.9
Statistical uncertainty	2	2
fs combs	0.1	0.1
Link uncertainty	< 0.1	0.03
Counter synchronization*	10	< 0.01
Gravity potential correction**	0.4	0.4
Total clock comparison	**11.2**	**5.0**



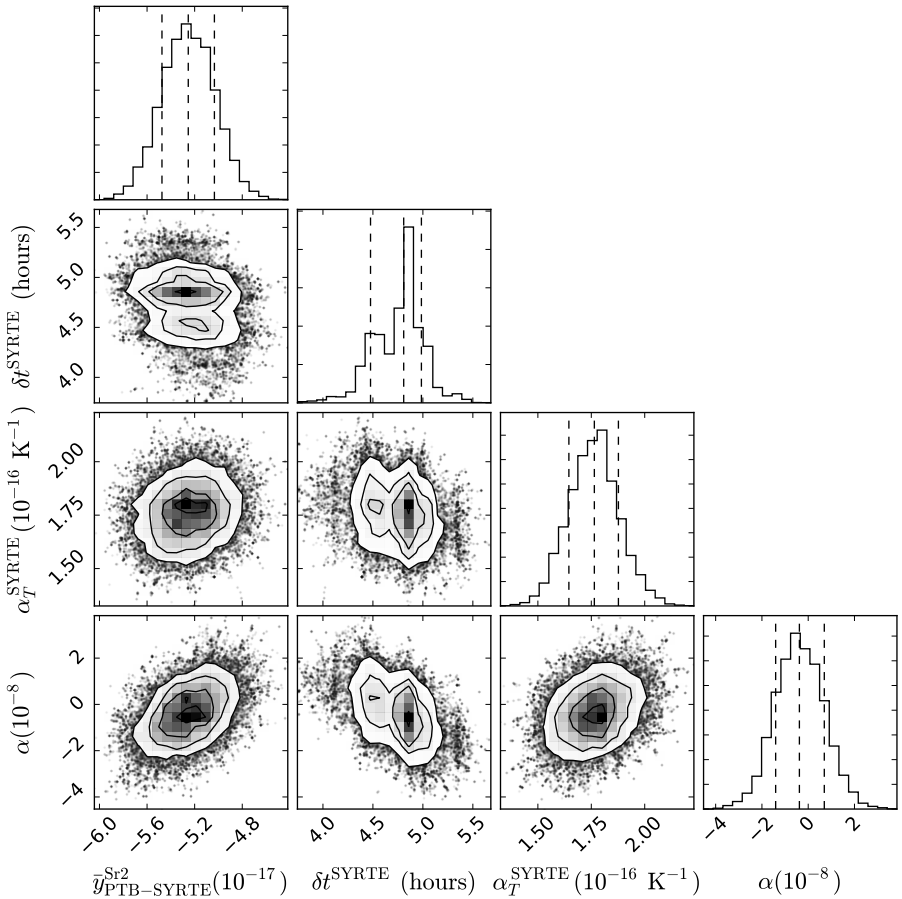
Statistical uncertainty 2×10^{-17} after $\simeq 1$ hour 150 hours of data

$$Sr_{PTB}/Sr_{SYRTE} - 1 = (4.7 \pm 5.0) \times 10^{-17}$$

C. Lisdat *et al.*, Nature Comm. **7** 12443 (2016)

The Sr clocks PTB-SYRTE comparison

$\delta t^{\mathrm{SYRTE}}$ (hours)	$\alpha_T^{\mathrm{SYRTE}}$ (10^{-16} K^{-1})	α (10^{-8})
4.81 ± 0.25	1.76 ± 0.12	-0.38 ± 1.06



- Systematic effect with one day period
- A detailed analysis has shown it is most probably due to temperature variations in the SYRTE clock room
- Model of the effect of temperature variations fitted with the RMS model:

$$y_T(t) = \alpha_T \left[T(t - \delta t) - \bar{T} \right]$$

Conclusion

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Test of Special Relativity Using a Fiber Network of Optical Clocks

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	Sr2/SrB SYRTE - Sr NPL combined	Sr2 SYRTE - Sr PTB
$\alpha(10^{-8})$	-2.83 ± 6.19	-0.38 ± 1.06

- Test limited by noise sources on the clock systems → reduction in uncertainty of more than one order of magnitude within reach
- Limitations: RMS is a non dynamical framework → use SME or dark matter models

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