Helicity coherence in binary neutron star mergers and nonlinear feedback

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"Helicity coherence in binary neutron star mergers and non-linear feedback"
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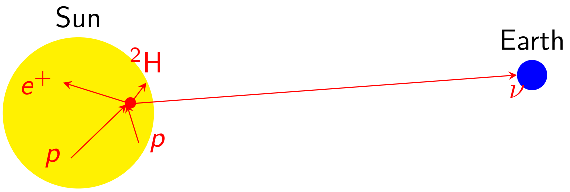
The solar neutrino problem



- Bethe, 1939 *pp* chain reaction H $\rightarrow^4$He, producing $> 99\%$ solar energy.
- 1960s Homestake : measure ν_e flux $\rightarrow$ deficit compared to Solar Standard Model prediction.

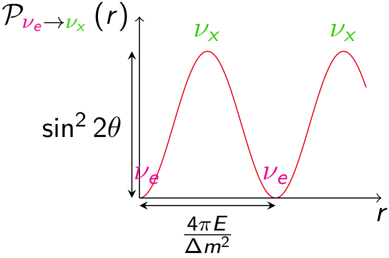
Where are these missing ν_es ?

Solving this problem : ν oscillations in vacuum

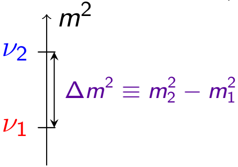
- Neutrinos are massive particles with mixing.

$$\underbrace{\begin{pmatrix} \nu_e \\ \nu_x \end{pmatrix}}_{\text{Flavor basis}} = \underbrace{\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}}_{\text{Mixing matrix } U} \underbrace{\begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}}_{\text{Mass basis}}$$

→ Interference.



$$\mathcal{P}_{\nu_e \to \nu_x}(r) = \sin^2 2\theta \sin^2 \left(\frac{\Delta m^2 r}{4E} \right)$$



Vacuum oscillations : spin formalism

- Density matrix formalism in the mean field approximation

$$\rho(r) = \begin{pmatrix} |\nu_e|^2 & \nu_e \nu_x^* \\ \nu_e^* \nu_x & |\nu_x|^2 \end{pmatrix} = \begin{pmatrix} \mathcal{P}_{\nu_e \to \nu_e}(r) & \times \\ \times & \mathcal{P}_{\nu_e \to \nu_x}(r) \end{pmatrix} \to \begin{matrix} i\dot{\rho} = [H, \rho] \\ i\dot{\bar{\rho}} = [\bar{H}, \bar{\rho}] \end{matrix}$$

- Decompose $\rho = \frac{1}{2}\left(\mathbb{1} + \vec{P} \cdot \vec{\sigma}\right)$:

$$\vec{P} = \mathrm{Tr} \rho \vec{\sigma}$$

$$\to P_z(t) = 2\mathcal{P}_{\nu_e \to \nu_e}(t) - 1.$$

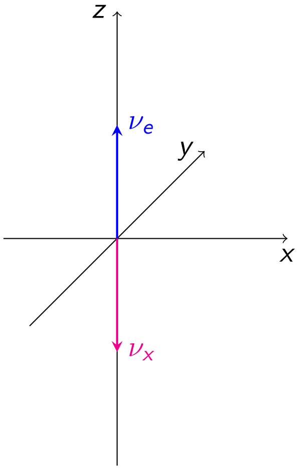
$$\to \dot{\vec{P}} = \vec{B} \times \vec{P}$$

Precession equation

- $H = \frac{1}{2}\left(\mathrm{Tr} H \mathbb{1} + \vec{B} \cdot \vec{\sigma}\right)$, $\vec{B} = \mathrm{Tr} H \vec{\sigma}$

Vacuum oscillations : Larmor precession

Flavor space



$$H_{\text{vac}}(E) = \frac{\Delta m^2}{4E}\begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix}$$

$$\Rightarrow \vec{B}_{\text{vac}}(E) = \frac{\Delta m^2}{2E}\begin{pmatrix} \sin 2\theta \\ 0 \\ -\cos 2\theta \end{pmatrix}$$

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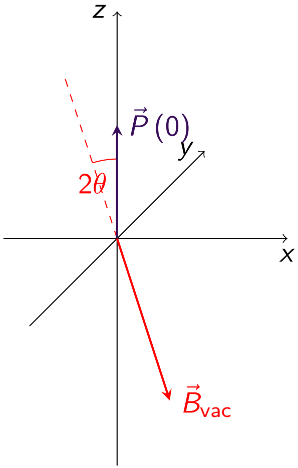

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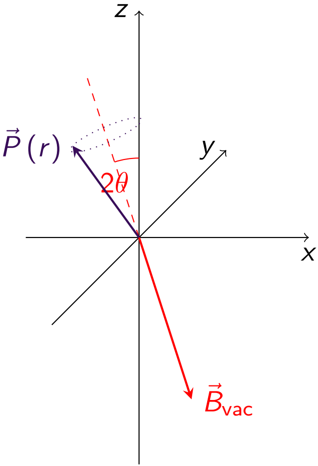

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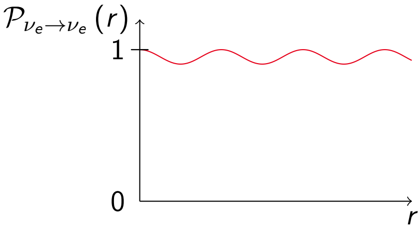
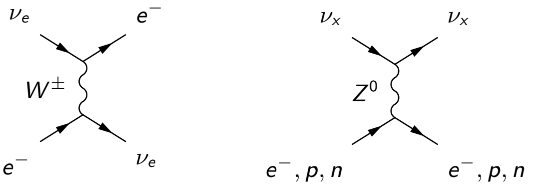


Neutrino in the Sun : Mikheev Smirnov Wolfenstein effect

[Wolfenstein, PRD17, 1978] [Mikheev, Smirnov, Sov. J. Nucl. Phys., 1985]

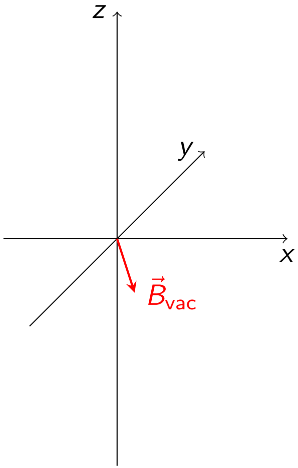


CC : Only ν_e.   NC : flavor blind.

- $H = H_{\text{vac}} \rightarrow H = H_{\text{vac}} + H_{\text{mat}}$, with $H_{\text{mat}}(r) = \begin{pmatrix} \sqrt{2} G_F n_e(r) & 0 \\ 0 & 0 \end{pmatrix}$.
- $\vec{B} = \vec{B}_{\text{vac}} \rightarrow \vec{B} = \vec{B}_{\text{vac}} + \vec{B}_{\text{mat}}$, with $\vec{B}_{\text{mat}}(r) = \begin{pmatrix} 0 \\ 0 \\ \sqrt{2} G_F n_e(r) \end{pmatrix}$.

Mikheev Smirnov Wolfenstein (MSW) Resonance

Flavor space



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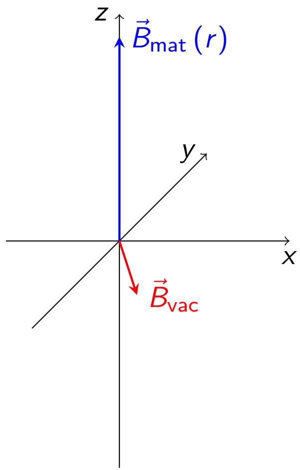

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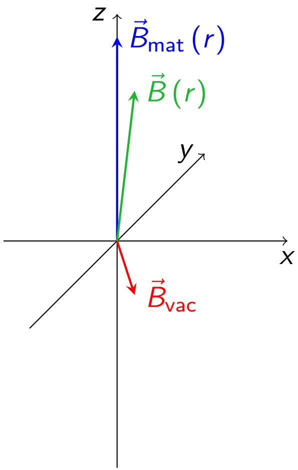

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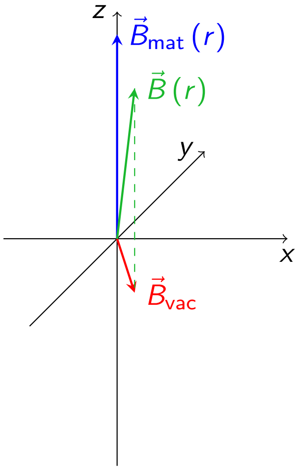

Mikheev Smirnov Wolfenstein (MSW) Resonance

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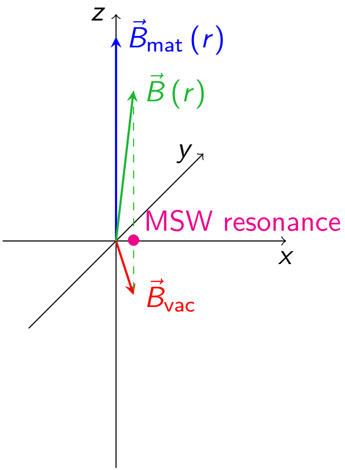
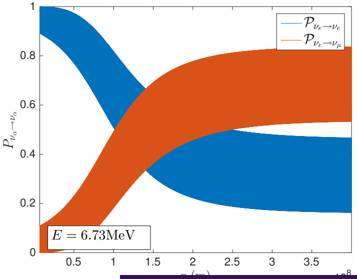


Resonance condition :

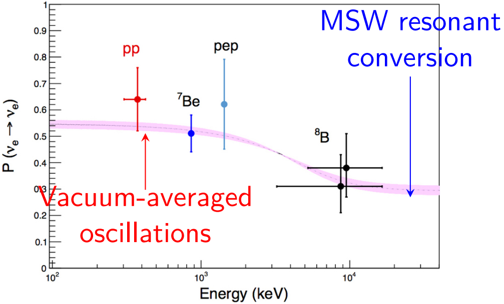
$$B_z \approx 0 \Leftrightarrow H_{11} - H_{22} \approx 0$$

$$H = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix}$$

If adiabatic : conversions.



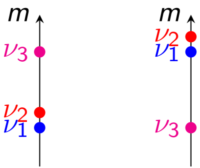
Solar problem solved !



[Borexino collaboration, Nature 512, 2014]

Open questions remain, eg

- Mass hierarchy



- Absolute mass scale
- Majorana or Dirac nature

Conversions in other astrophysical environments : Supernovae, Hypermassive Stars, Neutron Star (NS) - NS or NS-BH mergers ...

Beyond ?

$$i\dot{\rho} = [H, \rho] \qquad\qquad i\dot{\bar{\rho}} = [\bar{H}, \bar{\rho}]$$

- Most general equations in the mean field approximation : first order corrections to the relativistic limit $\propto m \rightarrow$ Helicity Coherence.

  [Volpe, Vaananen, Espinoza, PRD87, 2013]
  [Vlasenko, Cirigliano, Fuller, PRD89, 2014]
  [Serreau, Volpe, PRD90, 2014]

- Couples $\nu_L \leftrightarrow \nu_R$ (Dirac) or $\nu \leftrightarrow \bar{\nu}$ (Majorana).
- First study of this term in a toy model with only one Majorana neutrino flavor [Vlasenko, Fuller, Cirigliano, 2014] : significant conversions $\nu \leftrightarrow \bar{\nu}$.

$\rightarrow$ Can these corrections produce some effects in a more realistic scenario ?

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Theoretical framework

- Consider Majorana neutrinos.
- Corrections to the relativistic limit : matrices $2 \times 2 \rightarrow 4 \times 4$.

$$\rho \longrightarrow \rho_{\mathcal{G}} = \left(\begin{array}{c|c} \rho & \zeta \\ \hline \zeta^{\dagger} & \bar{\rho}^{T} \end{array}\right)$$

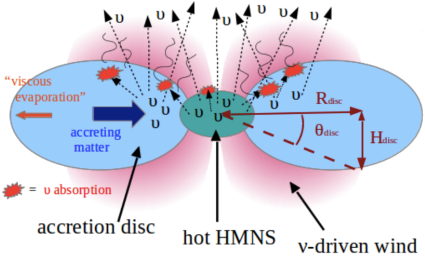
- ρ $(\bar{\rho})$: density matrices for ν $(\bar{\nu})$;
- ζ : coupling ν-$\bar{\nu}$ sectors.

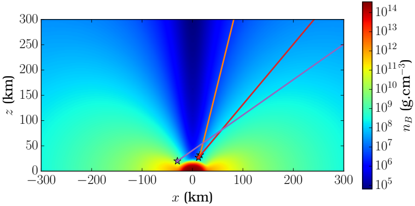
$$H \longrightarrow h_{\mathcal{G}} = \left(\begin{array}{c|c} H & \Phi \\ \hline \Phi^{\dagger} & -\bar{H}^{T} \end{array}\right)$$

- H $(\bar{H})$: Hamiltonian for ν $(\bar{\nu})$;
- Φ : coupling ν-$\bar{\nu}$ sectors, $\propto \frac{m}{E} \approx 10^{-8}$.

- $i\dot{\rho}_{\mathcal{G}} = [h_{\mathcal{G}}, \rho_{\mathcal{G}}]$ holds for the generalized matrices.
- $\Phi \rightarrow 0$: $i\dot{\rho} = [H, \rho]$ and $i\dot{\bar{\rho}} = [\bar{H}, \bar{\rho}]$.

Neutrino propagation in Binary Neutron Star mergers





[Perego et al., Mon.Not.Roy.Astron.Soc. 443, 2014]

- ν-driven winds : strong candidates for r-process nucleosynthesis.

$$\left.\begin{array}{l}\nu_e + n \to p + e^- \\ \bar{\nu}_e + p \to n + e^+\end{array}\right\} \text{ Set } Y_e = \frac{p}{n+p}.$$

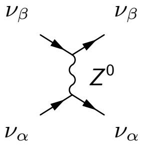
- Conversions $\to$ swap $\overset{(-)}{\nu}_e$, $\overset{(-)}{\nu}_x$ fluxes.

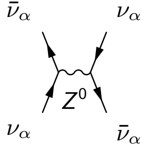
$\to$ It is crucial to understand flavor conversions.

ν sector, no $\nu \leftrightarrow \bar{\nu}$ coupling : neutrino self-interaction

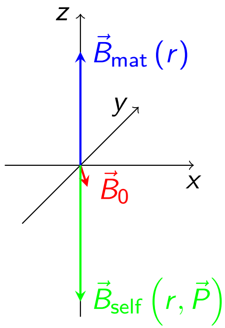
- Very high neutrino luminosities : self-interaction.

→ Introduce nonlinearity : $H \to H(\rho) = H_{\text{vac}} + H_{\text{mat}} + H_{\text{self}}(\rho)$.





Flavor space

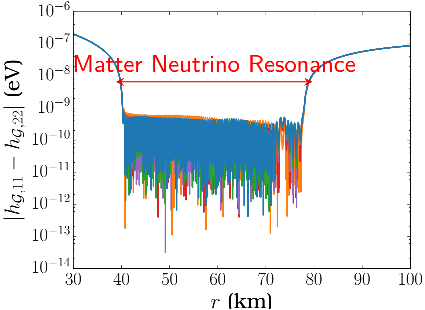


- $\vec{B} = \vec{B}_{\text{vac}} + \vec{B}_{\text{mat}} + \vec{B}_{\text{self}}$
- $L_{\bar{\nu}_e} > L_{\nu_e}$: $B_z^{\text{self}} < 0 \to$ possible MSW-like cancellation $B_z = H_{11} - H_{22} \approx 0$.

→ **Matter Neutrino Resonance.** [Malkus, Kneller, McLaughlin, Surman, PRD86,2012]

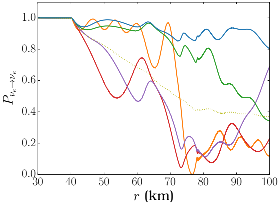
- Can be maintained over long distances because of a nonlinear feedback : conversions.

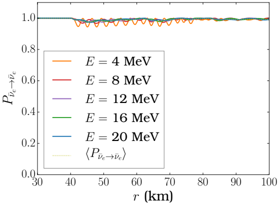
Matter Neutrino Resonance



$$H = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix}$$

Nonlinear feedback : resonance maintained because of non-linearity from neutrino self-interactions.

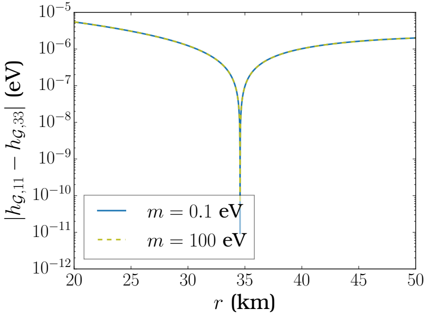




Helicity coherence : analytical and numerical results

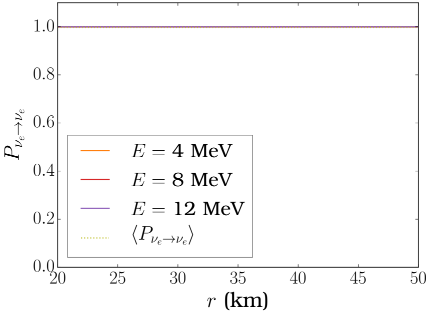
- $\frac{m}{E} \approx 10^{-8} \rightarrow$ Look for MSW-like resonance conditions that could enhance $\nu_e \leftrightarrow \bar{\nu}_e$ conversions, ie $h_{\mathcal{G},11} - h_{\mathcal{G},33} \approx 0$.

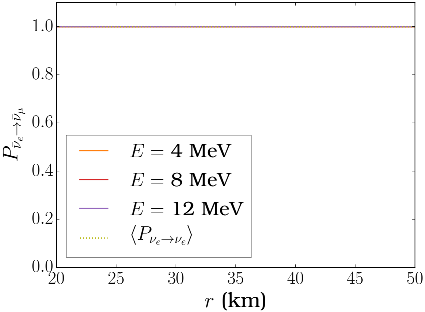
$$h_{\mathcal{G}} = \left[\begin{array}{c|c} H & \Phi \\ \hline \Phi^\dagger & -\bar{H}^T \end{array}\right]$$



- No nonlinear feedback : extremely narrow resonance.
- Artificially taking $m = 100$ eV : no difference.

Helicity Coherence : numerical results





$\hookrightarrow$ Run around the resonance : no conversions,
contrary to what was found in [Vlasenko, Fuller, Cirigliano, 2014] with $m = 1$ eV.

Why does nonlinear feedback occur ?

Observations :

- Matter Neutrino Resonance ($\nu_e \leftrightarrow \nu_x$) : nonlinearity creates multiple MSW-like resonances enhancing adiabacity.
- One-flavor toy model : the same thing occurs.

But :

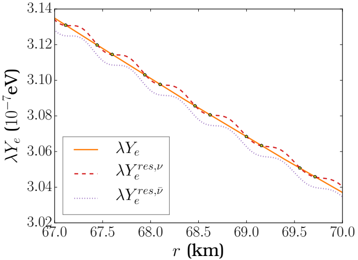
- Helicity coherence in our more realistic scenario : no nonlinear feedback, that could have widened the $\nu \leftrightarrow \bar{\nu}$ resonances.

Analysis :

- Nonlinear feedback $\leftrightarrow$ matching between matter and neutrino self-interaction derivatives.

Multiple MSW-like resonances

- Matter Neutrino Resonance : yo-yo effect between geometry and conversions $\rightarrow$ multiple MSW-like resonances.



- **Helicity coherence** : no such effect. Matching here : possible for very peculiar conditions.

True in other environments and for Dirac neutrinos.

- One flavor toy model [Vlasenko, Fuller, Cirigliano, 2014] : matter profile artificially smooth to enable the matching and the nonlinear feedback.

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Conclusion

- Neutrino flavor conversions in astrophysical environments : lots of on-going investigations, in particular for BNS.
- **Helicity coherence** in BNS mergers :
  - Non-relativistic corrections $\rightarrow$ coupling $\nu \leftrightarrow \bar{\nu}$, $\propto \frac{m}{E} = \mathcal{O}\left(10^{-8}\right)$.
  - Resonances possible in realistic astrophysical environments ...
  - ... but too narrow to enable $\nu \leftrightarrow \bar{\nu}$ conversions.
  - Analytical results : no widening due to non-linear feedback.

$\rightarrow$ **No effects appear due to non-relativistic corrections in a detailed astrophysical environment.**

- Open questions : role of collisions.

Conclusion

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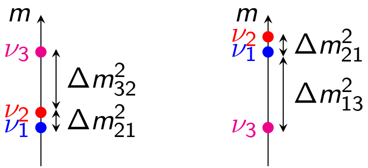
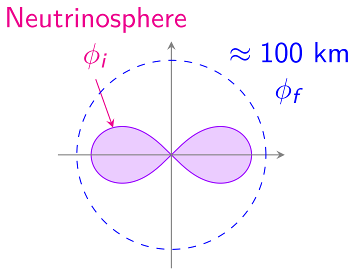
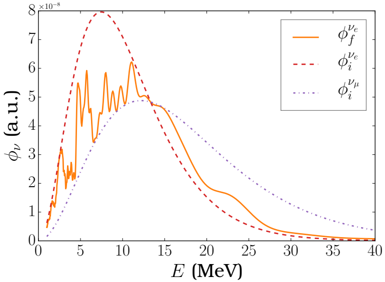
Thank you !



- $\mathcal{P}_{\nu_e \to \nu_x}(r) = \sin^2 2\theta \sin^2\left(\frac{\Delta m^2 r}{4E}\right)$ doesn't depend on $\mathrm{sign}\,(\Delta m^2)$.
- MSW resonance condition : $\frac{\Delta m^2}{2E}\cos 2\theta = \sqrt{2} G_F n_e(r) \rightarrow$ determine the sign of Δm^2_{21}.
- The sign of Δm^2_{32} or Δm^2_{13} still remains unknown.





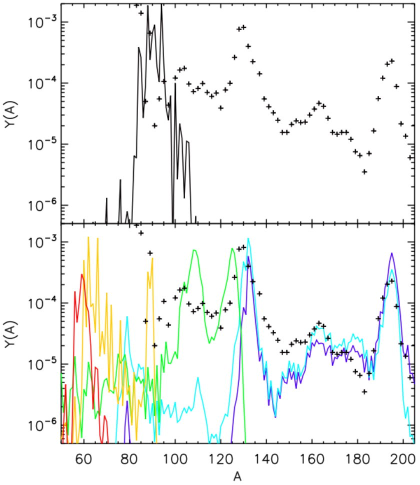


Figure: [Malkus, McLaughlin, Surman, PRD93, 2015]