Causal Nature and Dynamics of Trapping Horizons in Black Hole Collapse

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Reference:
arXiv: 1601.05109v2
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Introduction

- **Spherically symmetric** metric in comoving coordinates with ***t*** "cosmic time":

$$ds^2 = a^2(r,t)dt^2 + b^2(r,t)dr^2 + R^2(r,t)d\Omega^2$$

- **Proper time** and **proper distance** operators:

$$D_t \equiv \frac{1}{a}\frac{\partial}{\partial t} \;\Rightarrow\; U \equiv D_t R \qquad\qquad D_r \equiv \frac{1}{b}\frac{\partial}{\partial r} \;\Rightarrow\; \Gamma \equiv D_r R$$

- **Perfect Fluid:** $T^{\mu\nu} = (e+p)u^\mu u^\nu + pg^{\mu\nu}$

- Constraint equation (integrating G_{00}) : $\boxed{\Gamma^2 = 1 + U^2 - \frac{2M}{R}}$

- Mister-Sharp Mass : $M = \int 4\pi e R^2 dR$

$$D_t M = -4\pi p R^2 U$$

Trapping Horizons

Expansion of **ingoing/outgoing** null-rays :

$$k^a / l^a = \left(\frac{1}{a}, \pm\frac{1}{b}, 0, 0\right) \implies \theta_\pm = h^{cd}\nabla_c k_d = \frac{2}{R}(U \pm \Gamma)$$

$$h_{ab} = g_{ab} + \frac{1}{2}(k_a l_b + l_a k_b) \qquad k^a l_a = -2$$

Black Hole / Cosmological horizon : $\theta_\pm = 0 \Rightarrow \left.\frac{1}{a}\frac{dR}{dt}\right|_\pm \Rightarrow \Gamma^2 = U^2$

$$\boxed{R = 2M}$$

The horizon condition is independent of the slicing and holds also within a non-vacuum moving medium

The so-called **apparent horizon** of a black hole (which is a future trapping horizon) is the **outermost trapped surface for outgoing radial null rays** while the **trapping horizon for an expanding universe** (which is a past trapping horizon) **is foliated by the innermost anti-trapped surfaces for ingoing radial null rays**.

Causal Nature

$$\alpha \equiv \frac{\mathcal{L}_v\theta_v}{\mathcal{L}_{nv}\theta_v} \quad \begin{cases} \alpha > 0 & : \text{ space-like} \\ \alpha = 0\,/\infty & : \text{ null} \\ \alpha < 0 & : \text{ time-like} \end{cases}$$

Lie Derivatives:
$$\begin{cases} \mathcal{L}_+\theta_v = \mathcal{L}_k\theta_v = k^a\partial_a\theta_v = \left(\frac{1}{a}\frac{\partial}{\partial t} + \frac{1}{b}\frac{\partial}{\partial r}\right)\theta_v \\ \mathcal{L}_-\theta_v = \mathcal{L}_l\theta_v = l^a\partial_a\theta_v = \left(\frac{1}{a}\frac{\partial}{\partial t} - \frac{1}{b}\frac{\partial}{\partial r}\right)\theta_v \end{cases}$$

$$\mathcal{L}_\pm\theta_v = (D_t \pm D_r)\,\theta_v$$

$$\boxed{\alpha = \left.\frac{4\pi R^2(e+p)}{1-4\pi R^2(e-p)}\right|_H}$$

Horizon Velocity

3-velocity of the horizon with respect the matter: $v_H \equiv \left(\frac{b}{a}\frac{dr}{dt}\right)_H$

$$\theta_v = 0 \quad \Rightarrow \quad D_t\theta_v + \frac{b}{a}\frac{dr}{dt}D_r\theta_v = 0$$

$$v_H \equiv -\frac{D_t\theta_v}{D_r\theta_v} \quad \Rightarrow \quad v_H = -\left.\frac{D_t\left(\Gamma^2 - U^2\right)}{D_r\left(\Gamma^2 - U^2\right)}\right|_H$$

$$v_H = -\left.\frac{\mathcal{L}_+\theta_v + \mathcal{L}_-\theta_v}{\mathcal{L}_+\theta_v - \mathcal{L}_-\theta_v}\right|_H \quad \Rightarrow \quad v_H = \pm\frac{1+\alpha}{1-\alpha}$$

$$\boxed{v_H = -\left.\frac{U}{\Gamma}\right|_H \left.\frac{1+8\pi R^2 p}{1-8\pi R^2 e}\right|_H}$$

$$\begin{cases} |v_H| > 1: & \text{space-like} \\ |v_H| = 1: & \text{null} \\ |v_H| < 1: & \text{time-like} \end{cases}$$

$$G_{\mu\nu} = 8\pi T_{\mu\nu} \qquad T_{\mu\nu} = (e+p)u_\mu u_\nu - p g_{\mu\nu}$$

COSMIC TIME

$$ds^2 = -a^2\, dt^2 + b^2\, dr^2 + R^2 d\Omega^2$$

$$D_t \equiv \frac{1}{a}\left(\frac{\partial}{\partial t}\right) \qquad D_r \equiv \frac{1}{b}\left(\frac{\partial}{\partial r}\right)$$

- Proper time / space derivative

$$U \equiv D_t R \qquad \Gamma \equiv D_r R$$

- 4-velocity & Lorentz factor

$$D_t U = -\left[\frac{\Gamma}{(e+p)} D_r p + \frac{M}{R^2} + 4\pi R p\right]$$

- Euler equation

$$D_t \rho = -\frac{\rho}{\Gamma R^2} D_r(R^2 U)$$

- Continuity equation

$$D_t e = \frac{e+p}{\rho} D_t \rho$$

$$D_t M = -4\pi R^2 p U$$

- Mass conservation

$$dM = aU\,dt + b\Gamma\,dr$$

$$D_r a = -\frac{a}{e+p} D_r p$$

- Lapse equation / pressure gradients

$$D_r M = 4\pi R^2 \Gamma e$$

$$\Gamma^2 = 1 + U^2 - \frac{2M}{R}$$

- Constraint equation

Equation of State

energy density: $e = \rho(1 + \epsilon)$

pressure: $p = (\gamma - 1)\rho\epsilon$

rest mass density

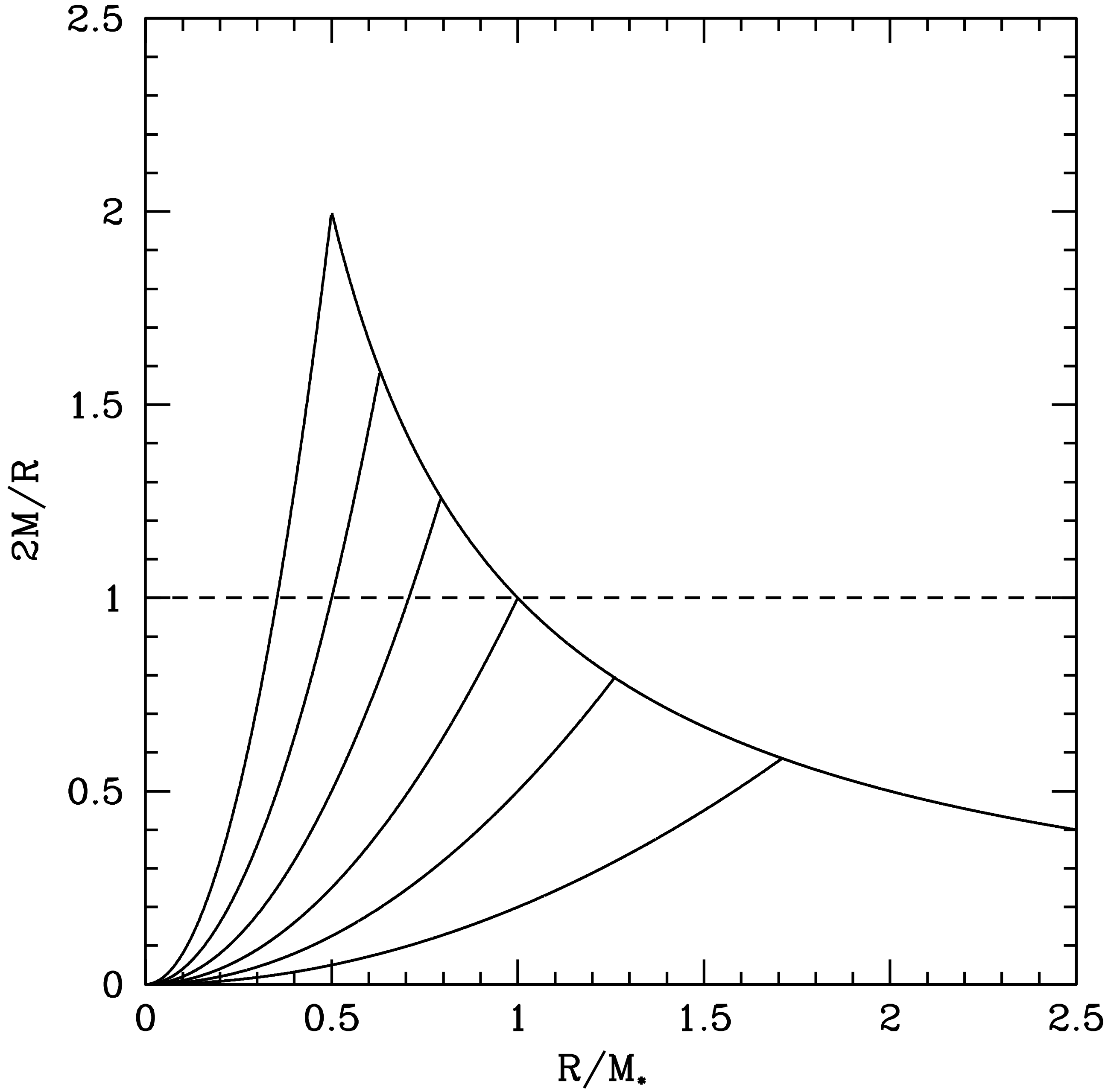
adiabatic index - particle degree of freedom

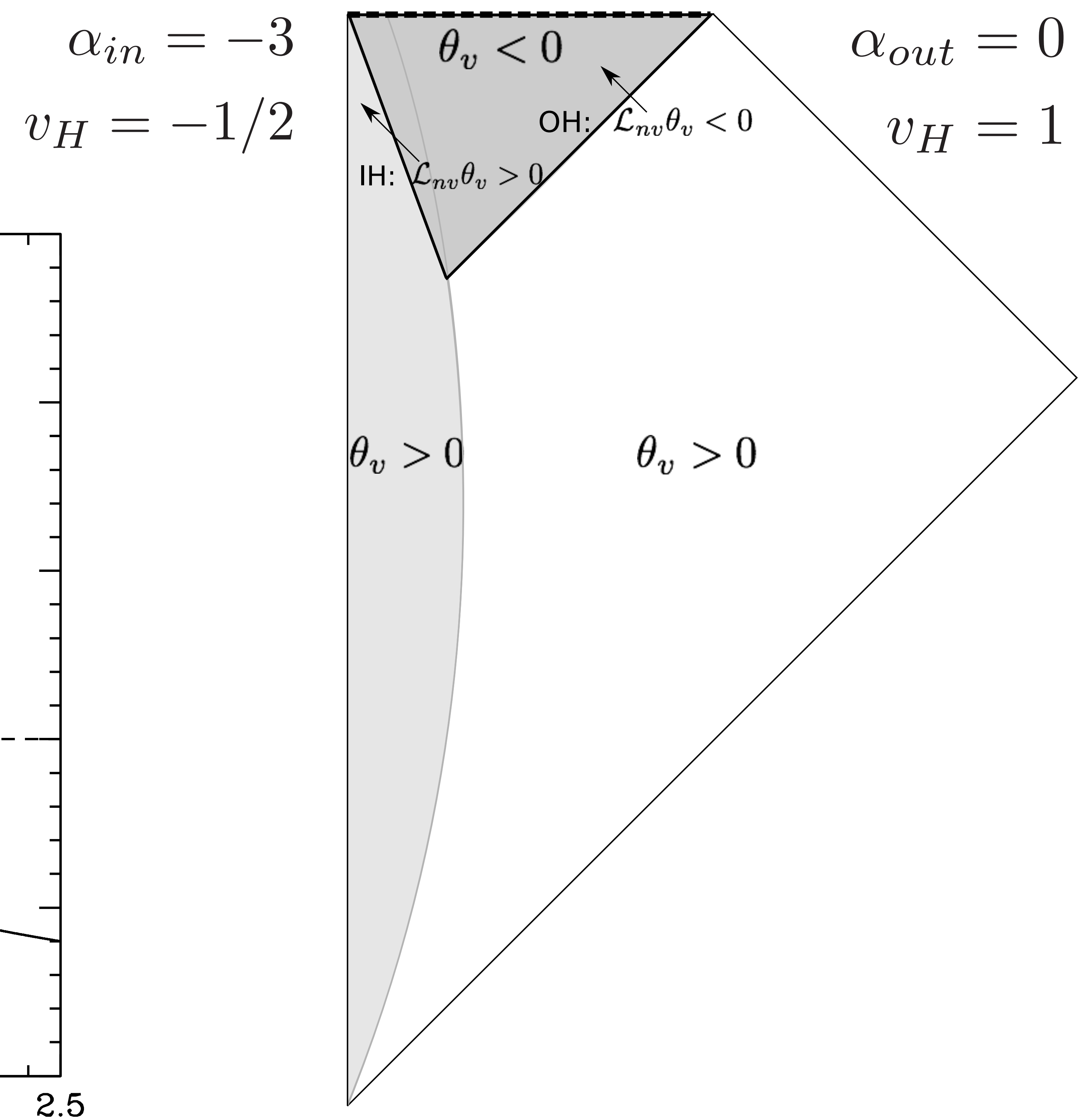
specific internal energy (velocity dispersion)

- Barotropic fluid (no rest mass density): $p = we$ with $w \in [0, 1]$
  - radiation dominated era: $w = 1/3$ RADIATION $(\gamma = 4/3)$
  - matter dominated era: $w = 0$ DUST $(\gamma = 1)$
- Polytropic fluid: $p = K(s)\rho^{\gamma}$ $(\gamma = 5/3,\ 4/3,\ 2)$
  - If the fluid is adiabatic (no entropy change): $K(s) = K$ (constant)

Oppenheimer-Snyder collapse

$$\frac{2M}{R} = \frac{8}{3}\pi R^2 e$$

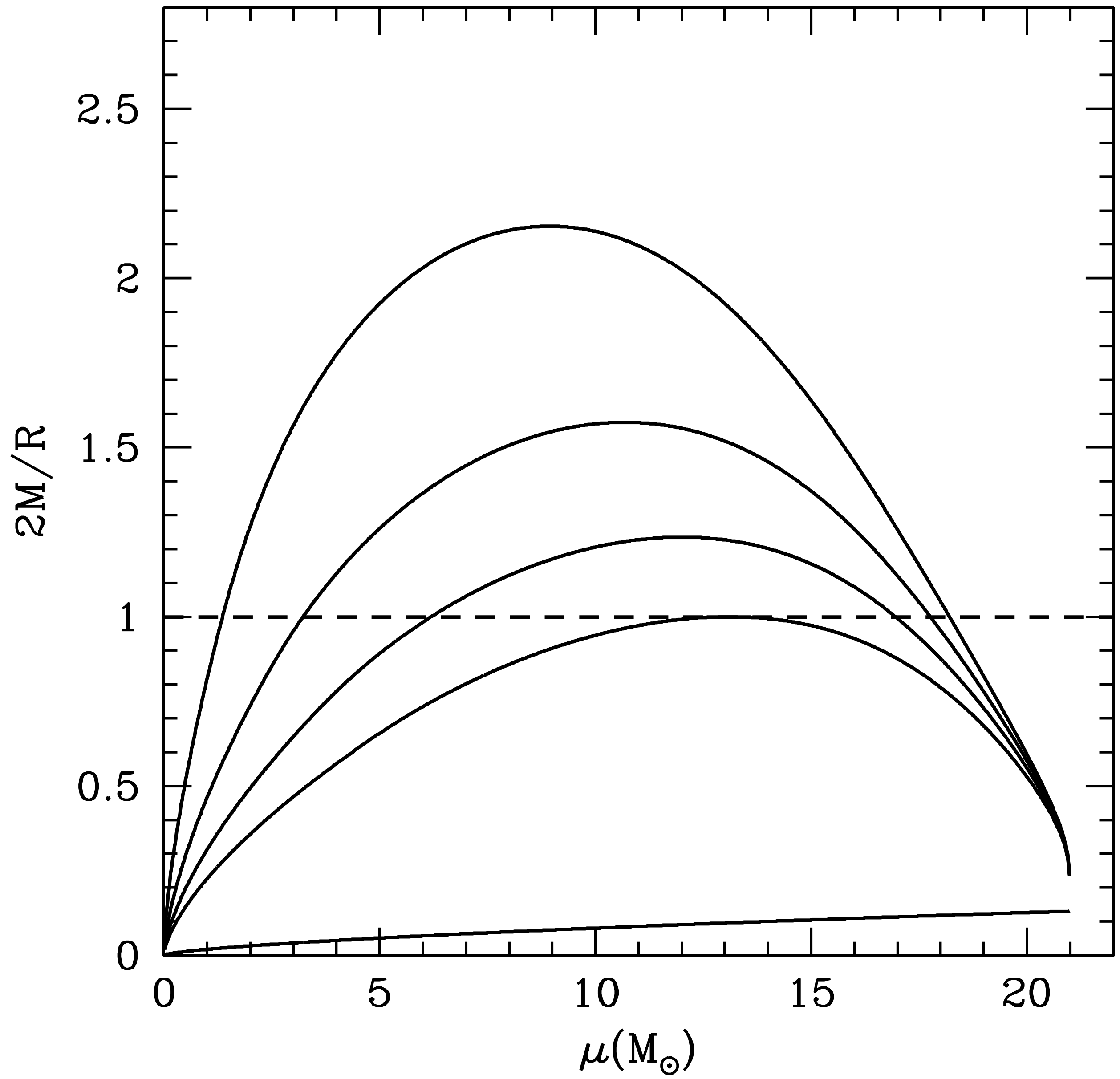


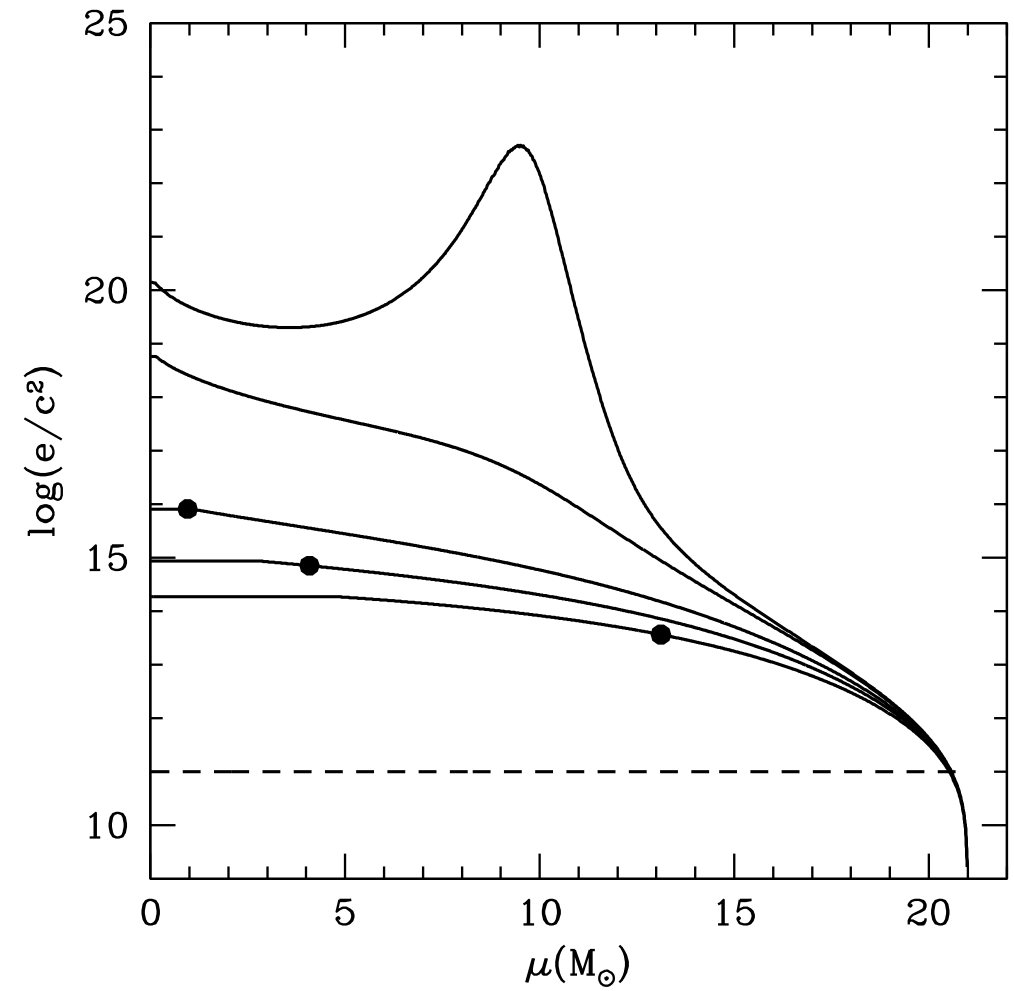


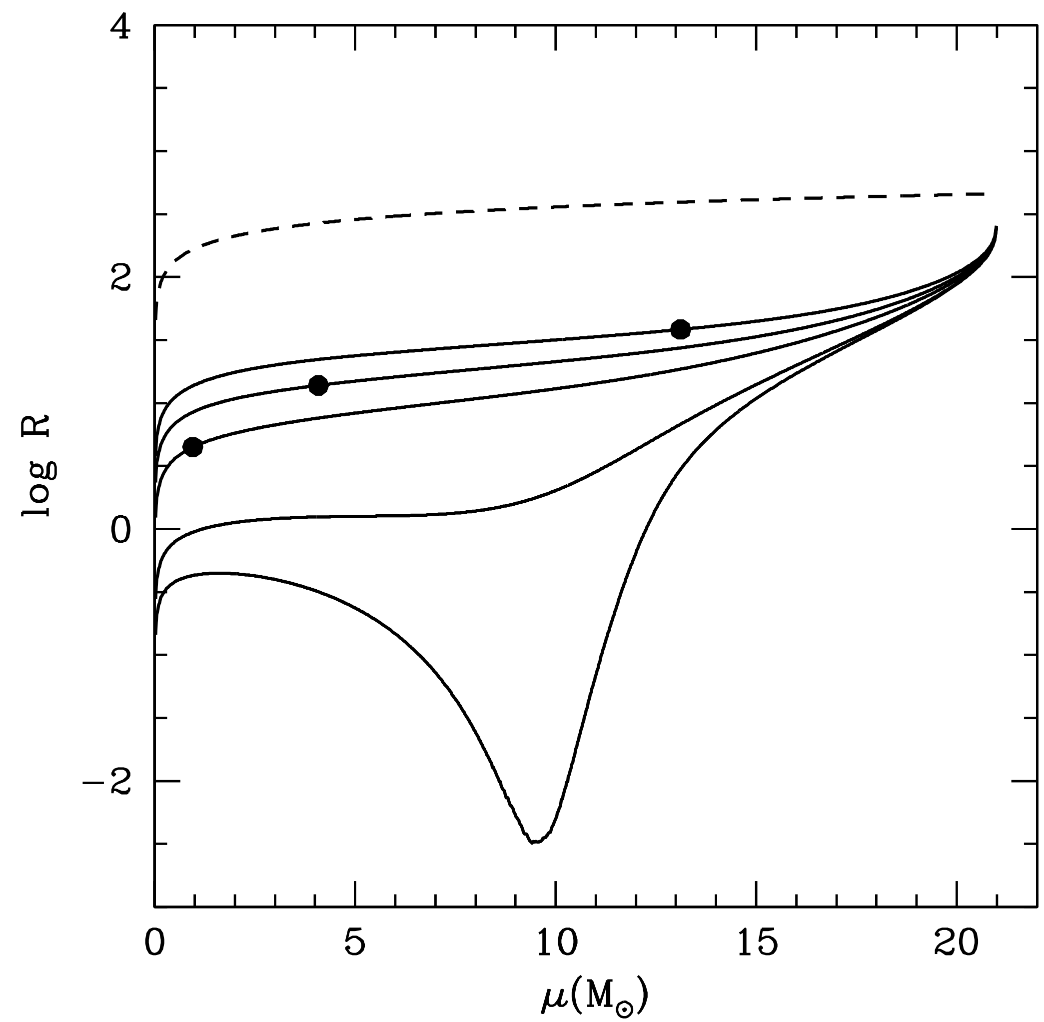
May & White Numerical Collapse (1966):

Homogenous initial density

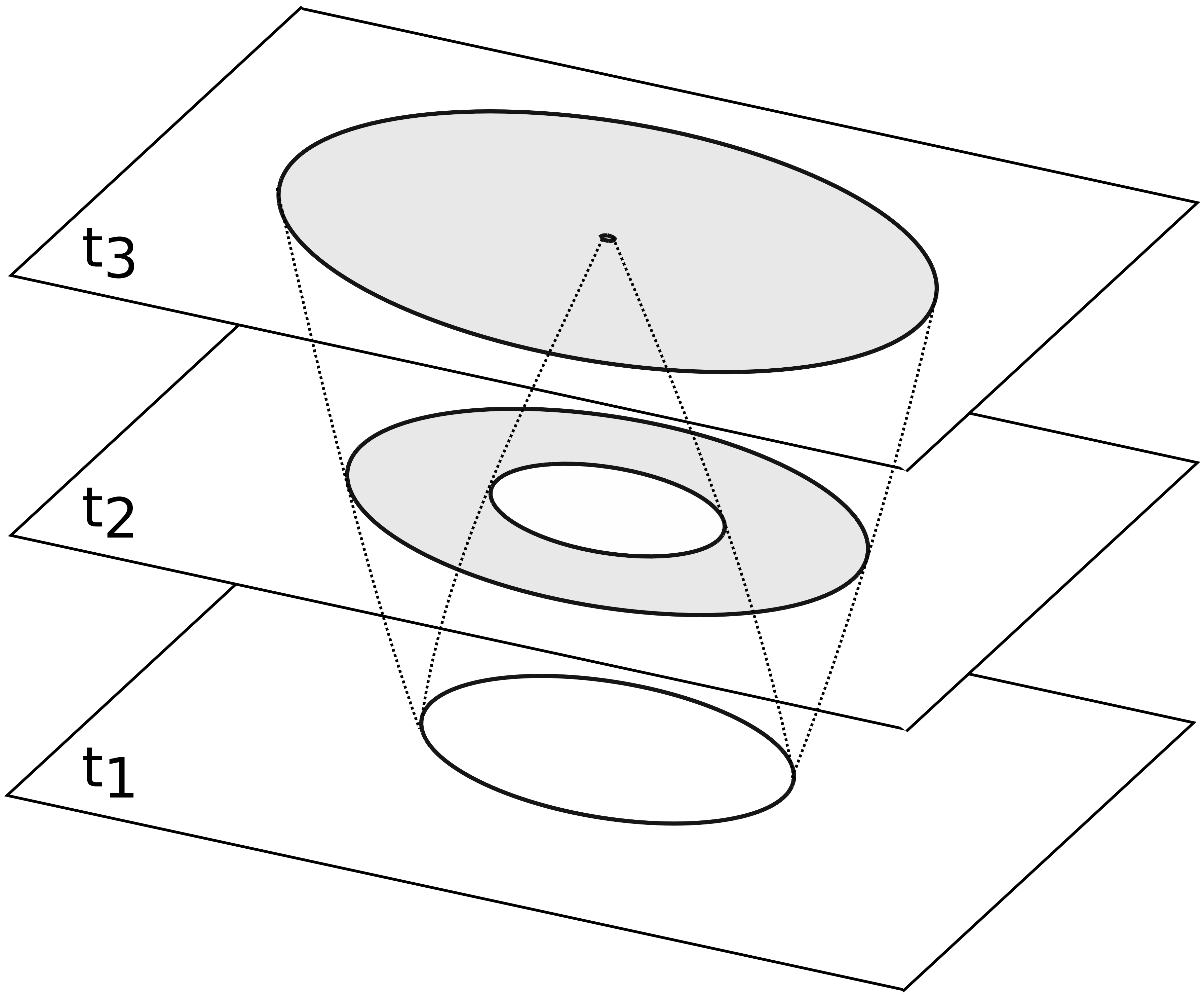
$$p = K\rho^{\gamma} \quad (\gamma = 5/3, K = 0.05)$$







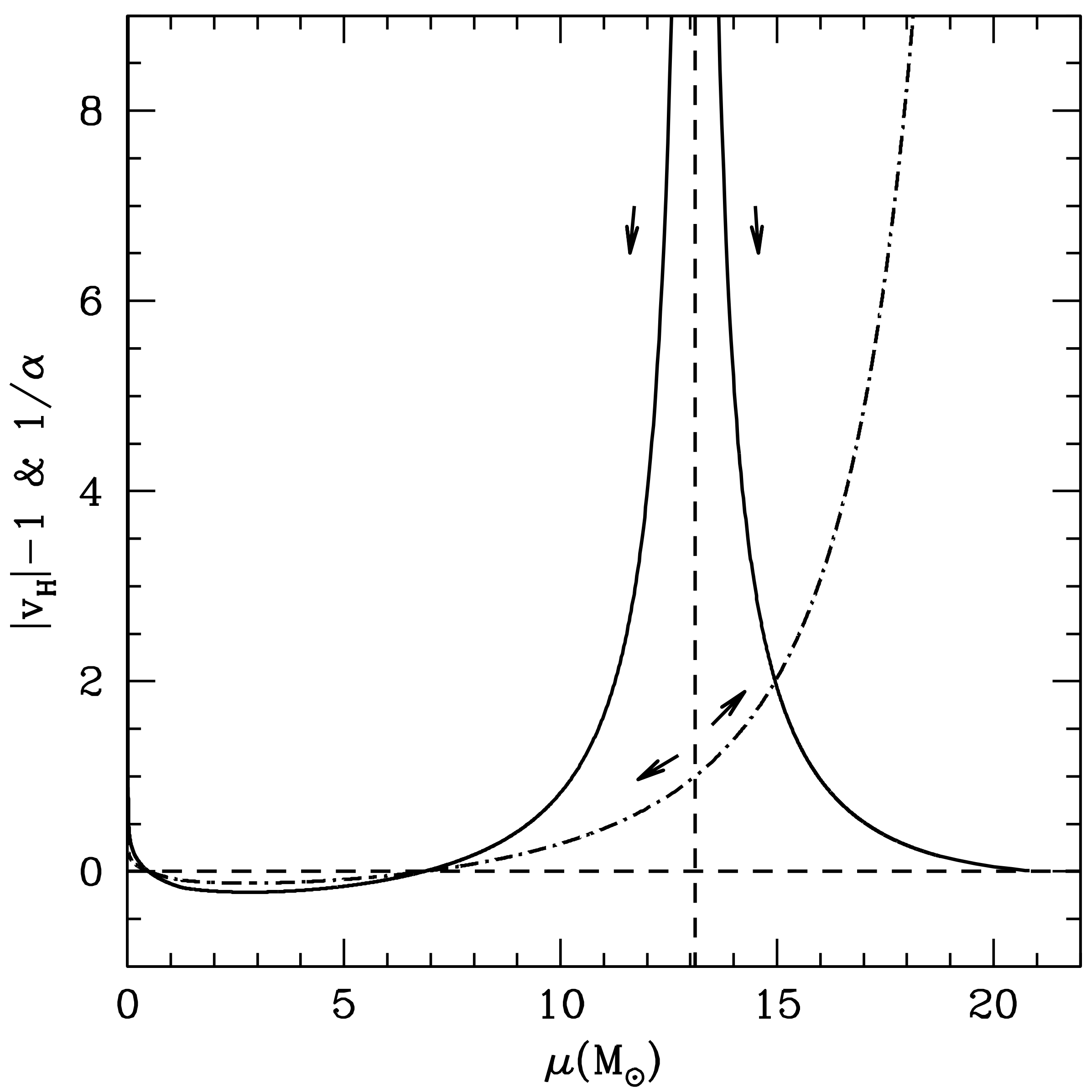
General scheme for in/out-going horizon evolution

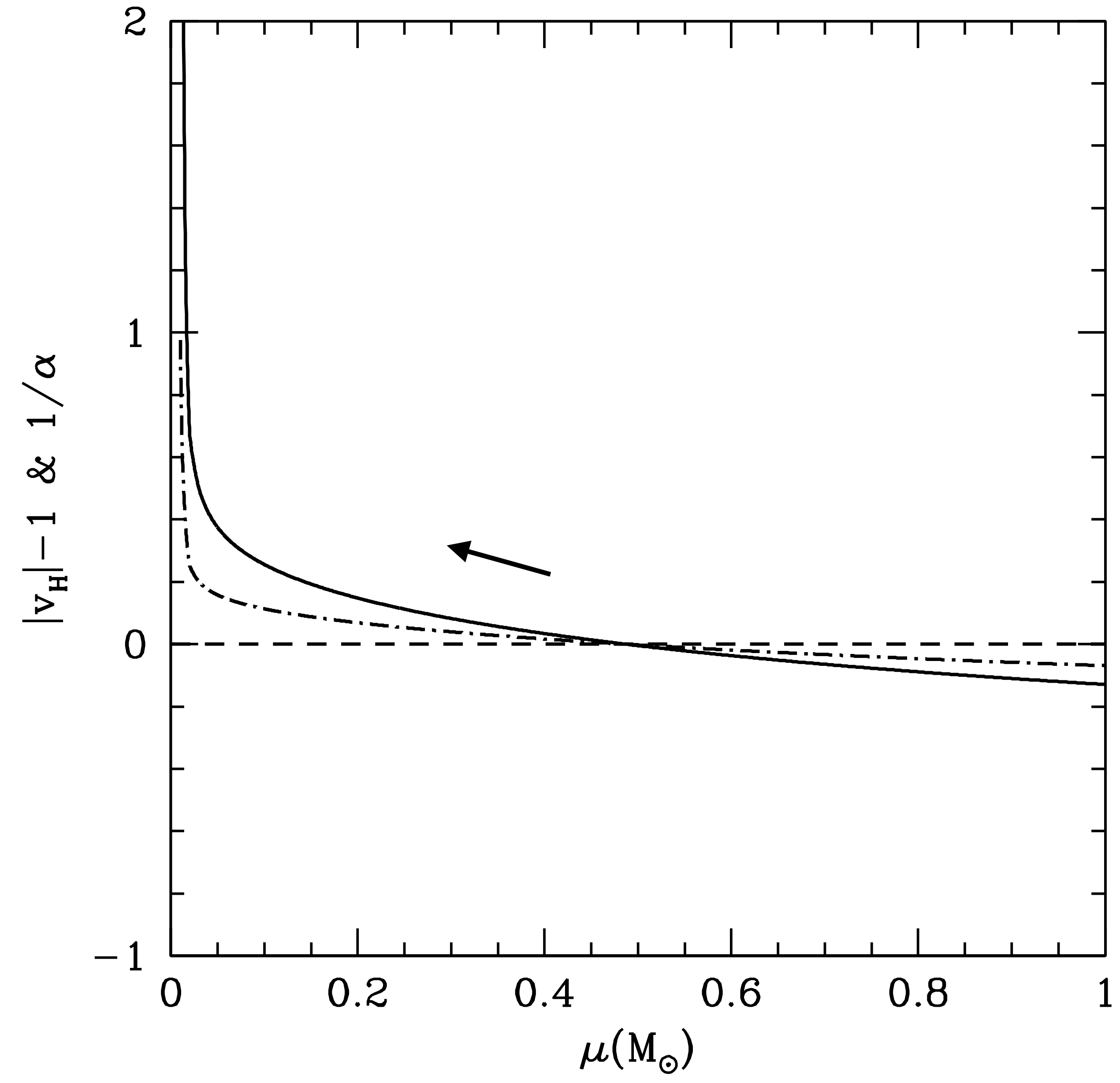


$$p = K\rho^{\gamma}\ (\gamma = 5/3, \text{HOM I.C.})$$

$$\boxed{\alpha = \frac{4\pi R_H^2(e+p)}{1 - 4\pi R_H^2(e-p)}}$$

$$\boxed{v_H = \frac{1 + 8\pi R_H^2 p}{1 - 8\pi R_H^2 e}}$$

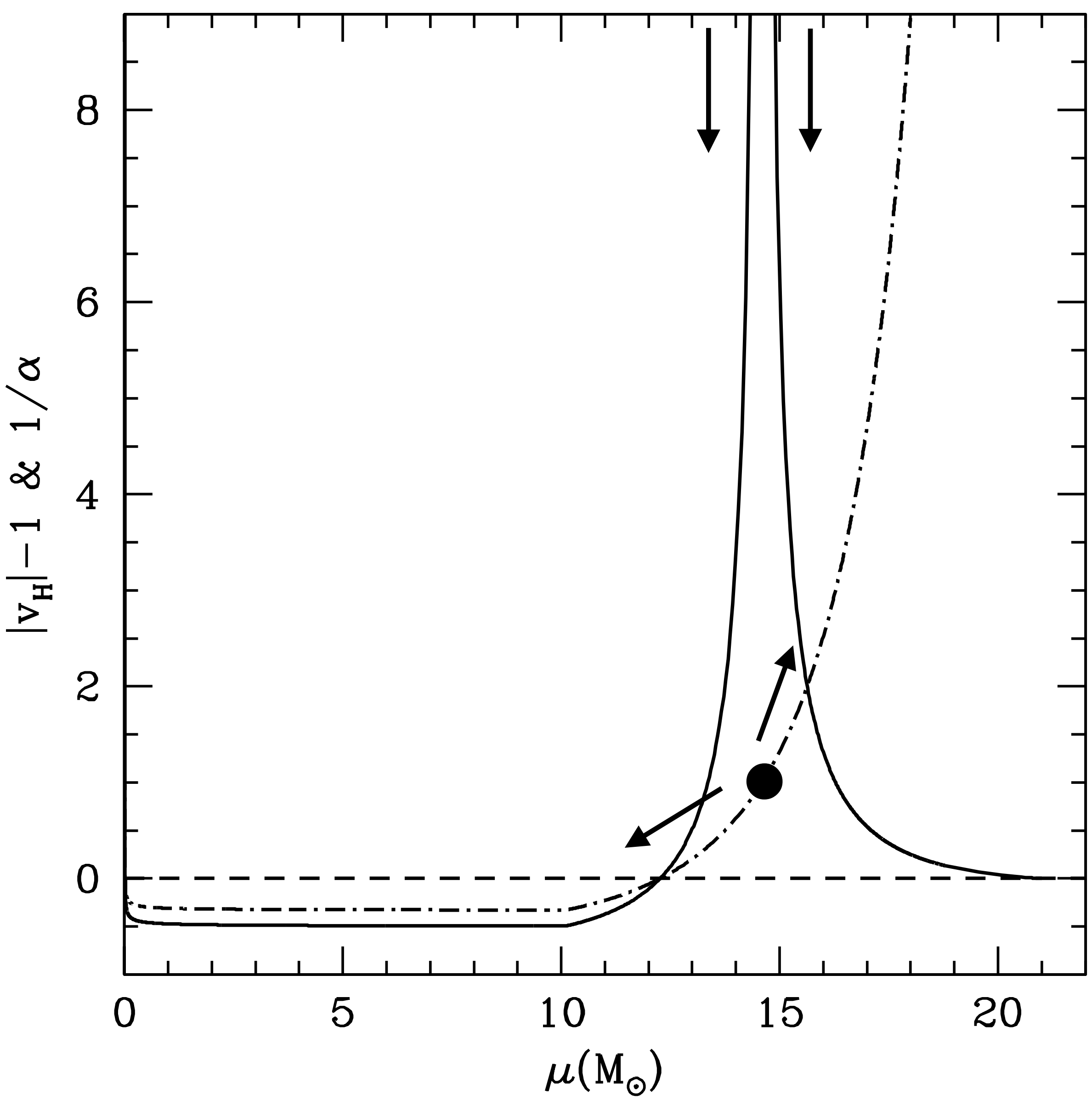


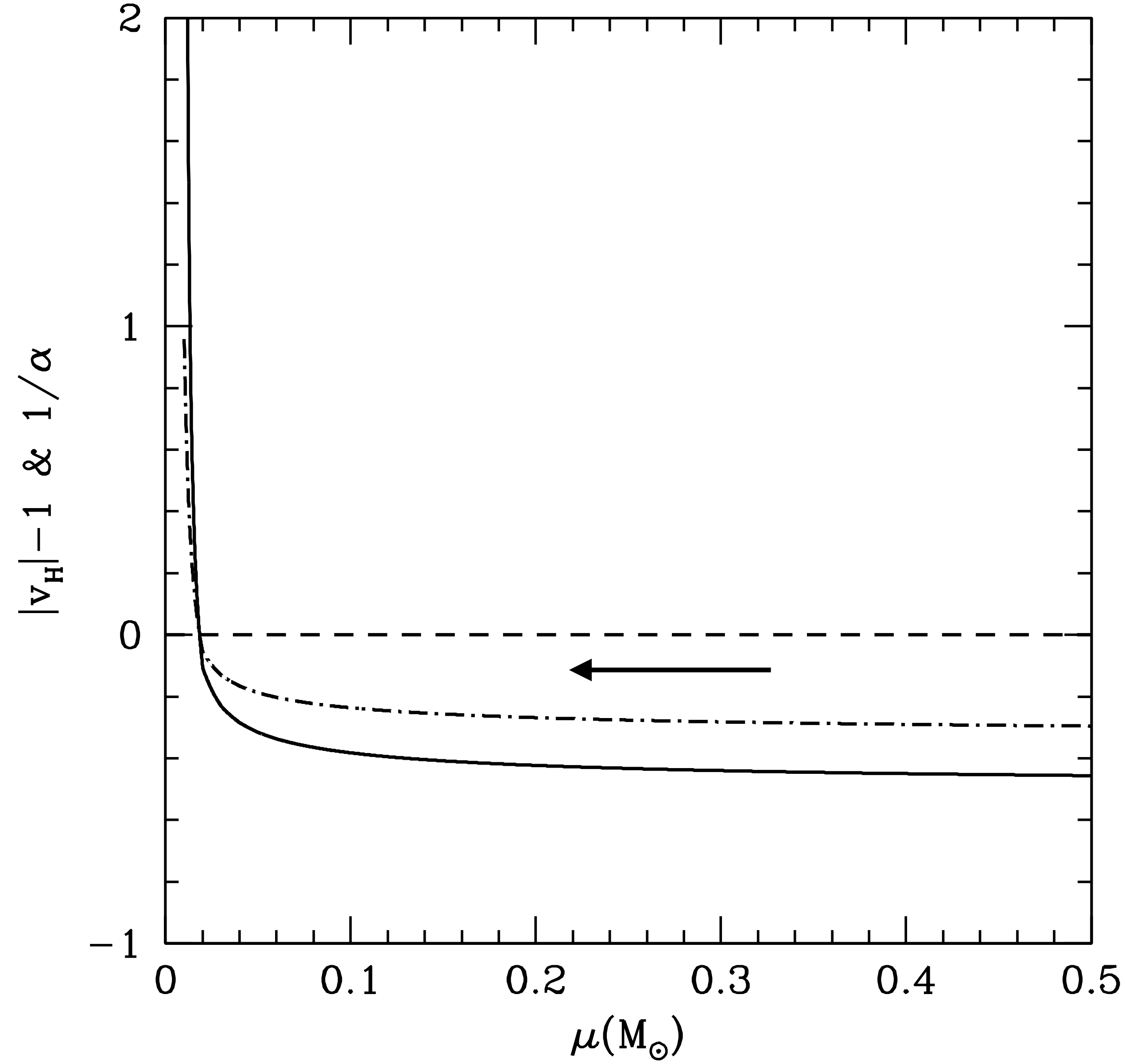


$$p = K\rho^{\gamma} \quad (\gamma = 4/3, \text{ HOM I.C.})$$

$$\boxed{\alpha = \frac{4\pi R_H^2 (e+p)}{1 - 4\pi R_H^2 (e-p)}}$$

$$\boxed{v_H = \frac{1 + 8\pi R_H^2 p}{1 - 8\pi R_H^2 e}}$$

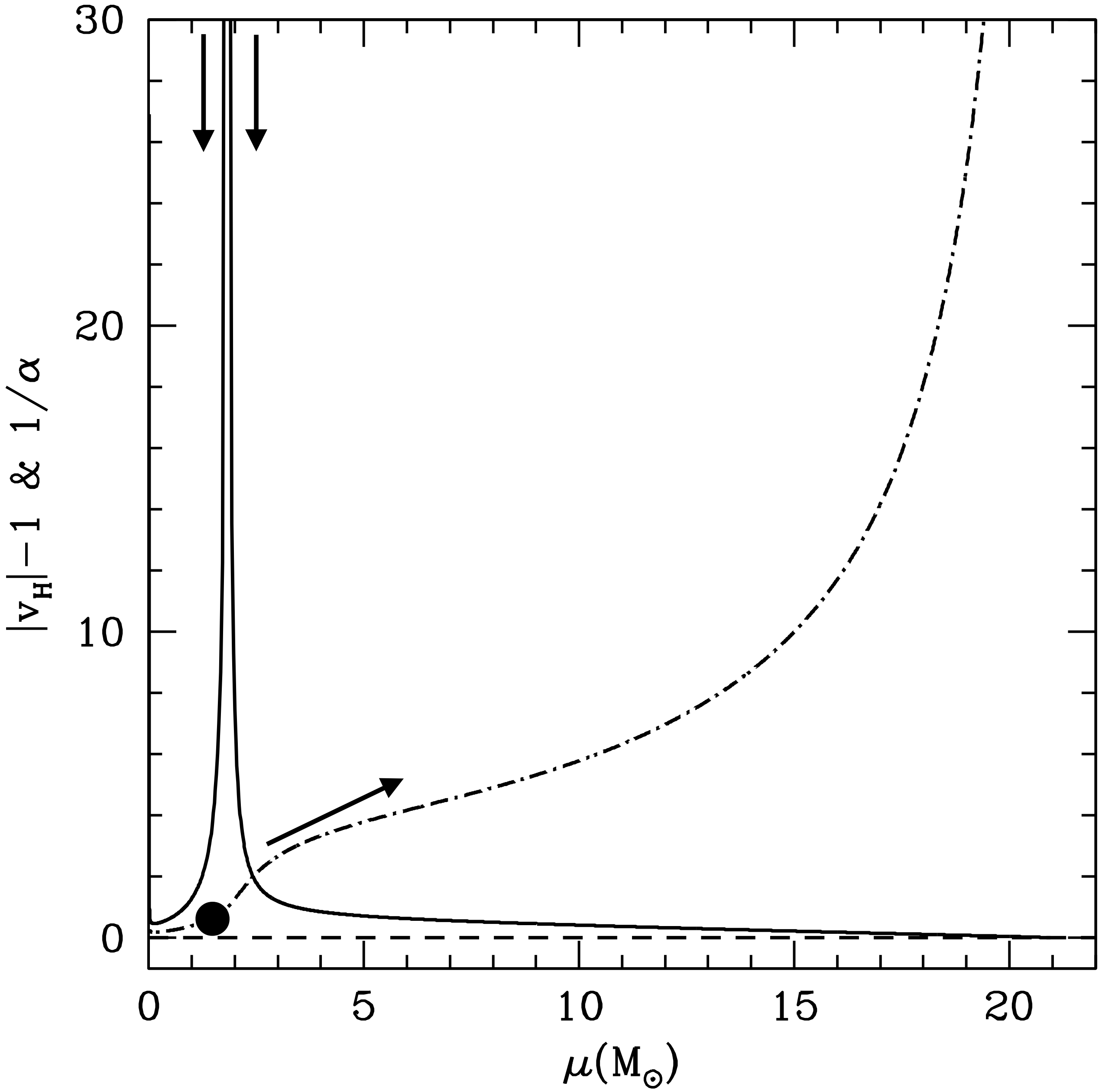


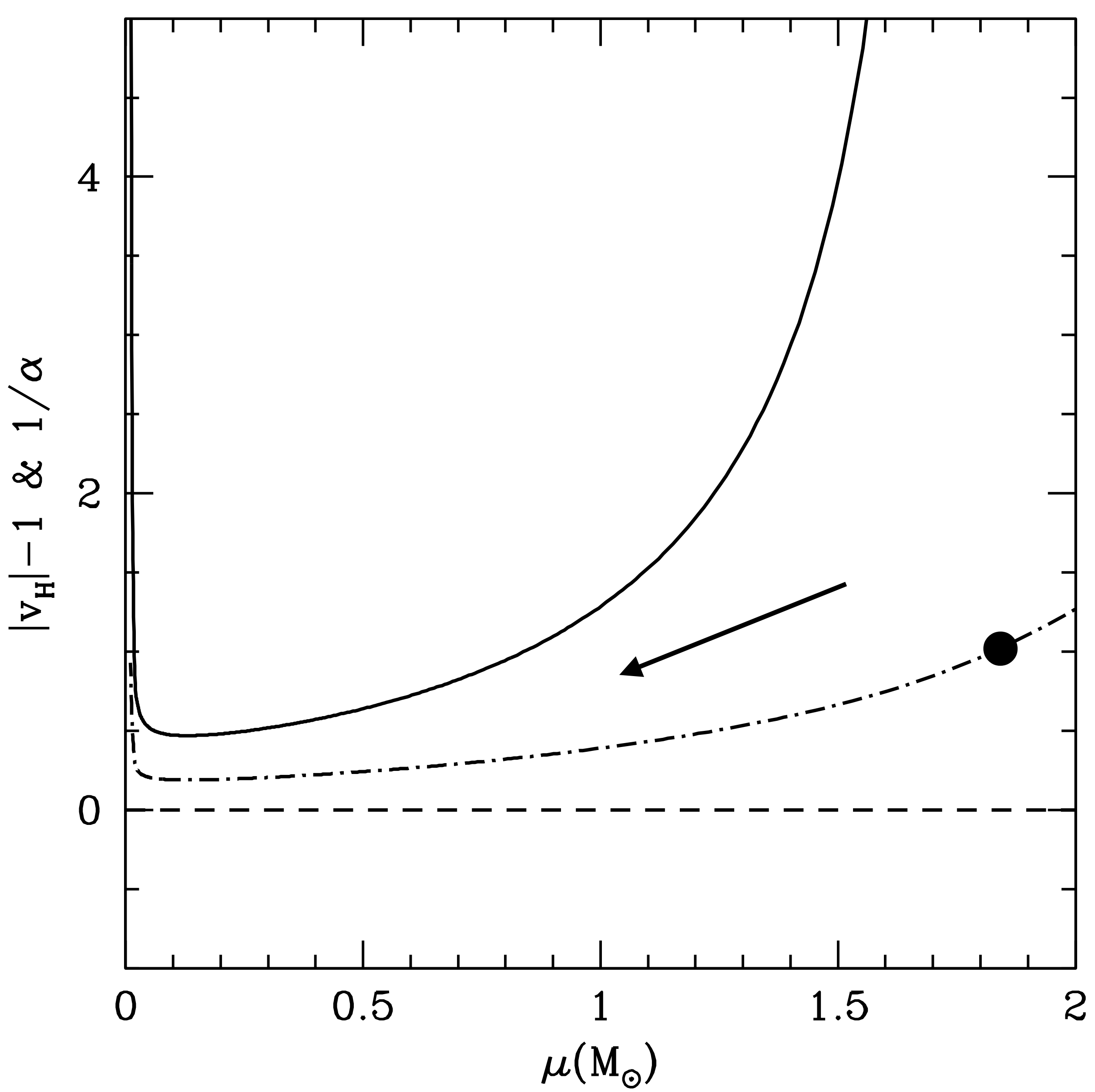


$$p = K\rho^{\gamma} \quad (\gamma = 5/3, \text{TOV I.C.})$$

$$\boxed{\alpha = \frac{4\pi R_H^2(e+p)}{1 - 4\pi R_H^2(e-p)}}$$

$$\boxed{v_H = \frac{1 + 8\pi R_H^2 p}{1 - 8\pi R_H^2 e}}$$



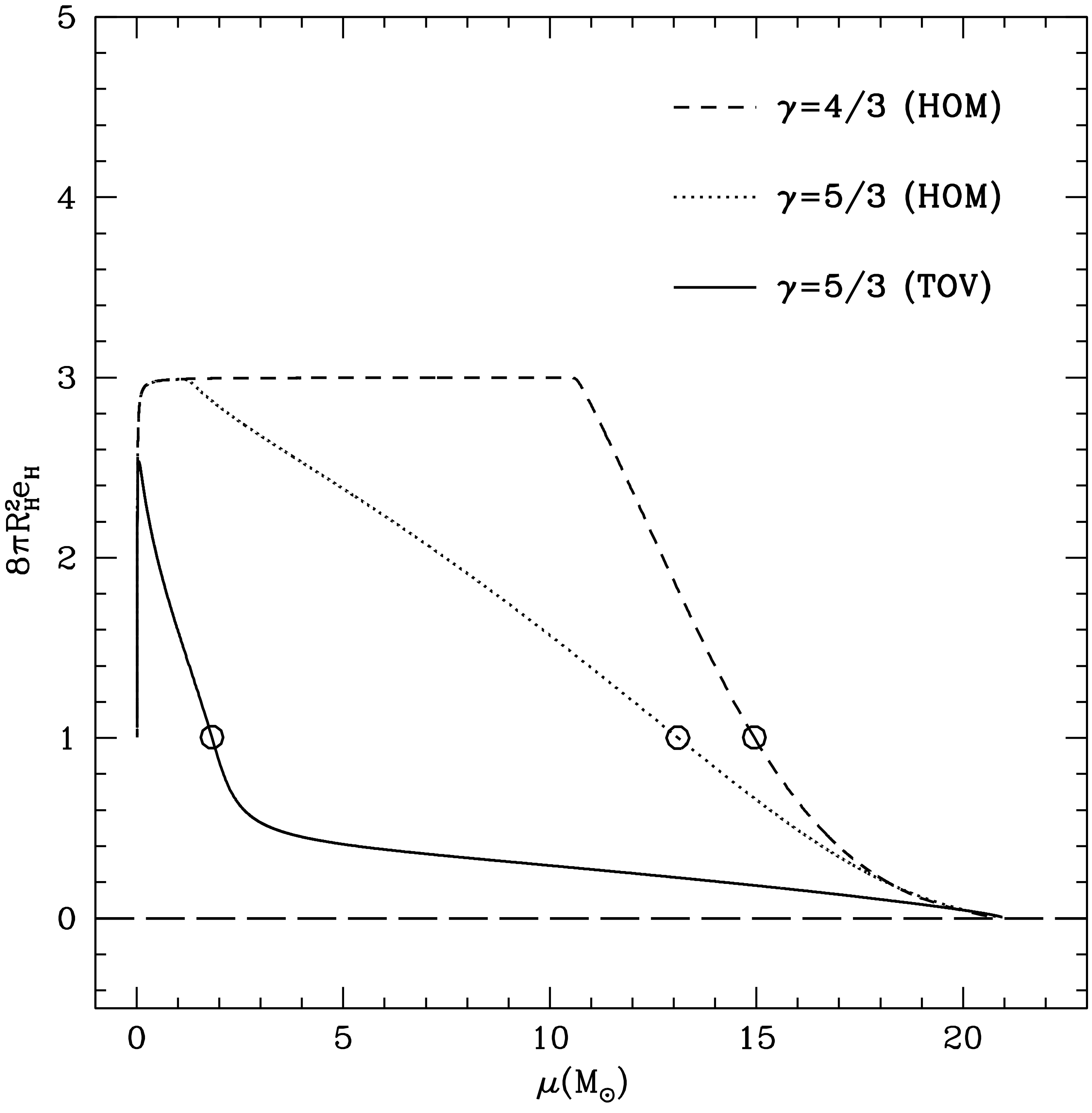


Causal Nature Summary

$$\boxed{\alpha = \frac{4\pi R_H^2(e+p)}{1-4\pi R_H^2(e-p)}}$$

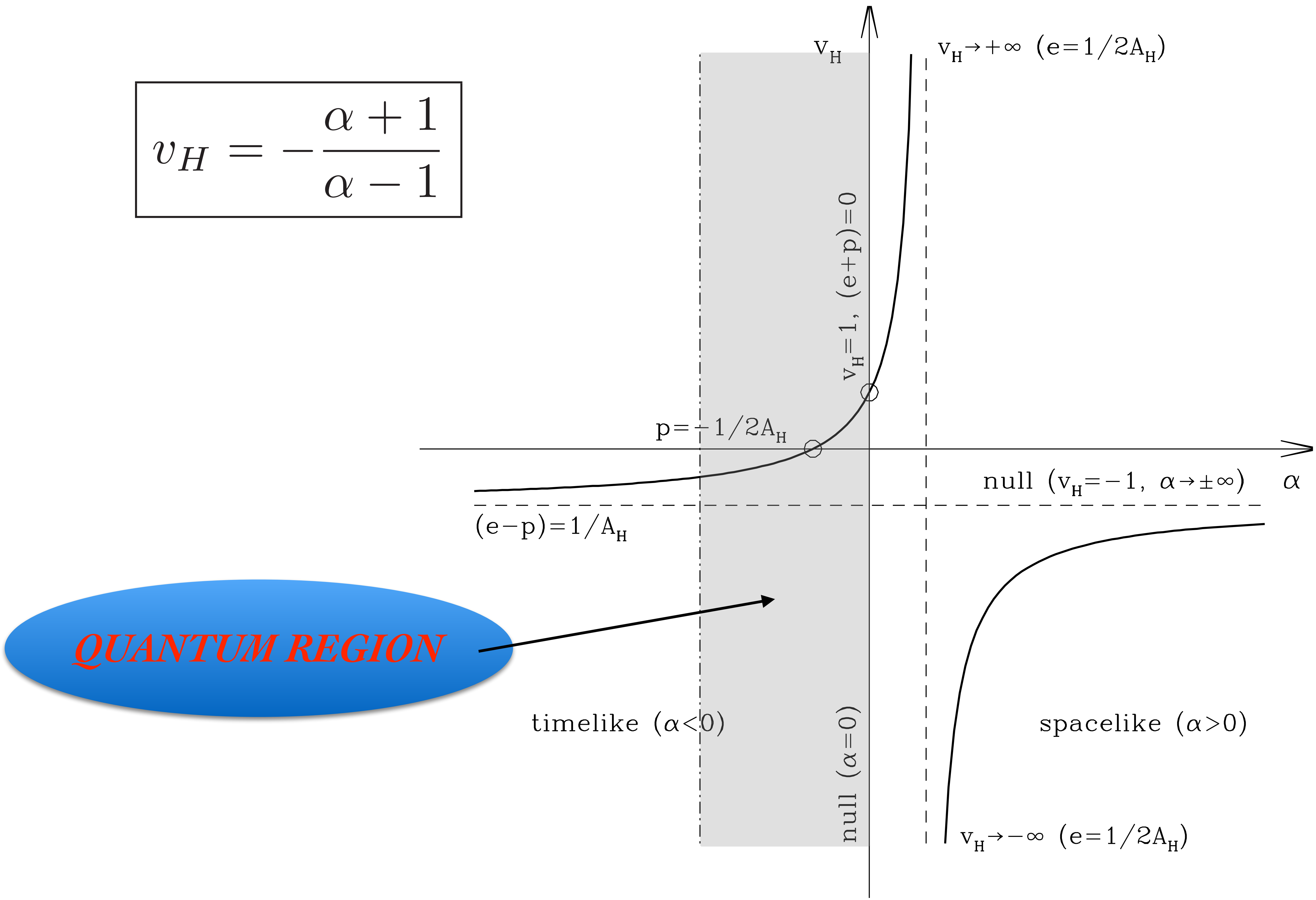
$$\boxed{v_H = \frac{1+8\pi R_H^2 p}{1-8\pi R_H^2 e}}$$

$$\boxed{(\alpha = 1) \Rightarrow \ e = \frac{1}{2A_H}}$$



Black Hole Horizon - Phase Diagram

$$v_H = -\frac{\alpha + 1}{\alpha - 1}$$



Conclusions & Future perpectives

- With the Misner-Sharp equations (cosmic time slicing) we have studied the causal nature of trapping horizons appearing in gravitational collapse for polytropic stars forming black holes using a spherically symmetric Lagrangian numerical code.

- Within the classical regime of GR we have observed space-like outgoing horizons and space-like/time-like ingoing horizons depending on the choice of the equations of state and initial conditions. **Pressure seems to play a key role!**

- The conditions of horizon formation and annihilation are independent of the initial conditions:

$$\alpha = 1\,,\ \ v_H = \pm\infty$$

- The formalism developed seems to show the possibility of incorporating quantum effects within the classical formulation of the GR-hydro equations modifying the equation of state accordingly to quantum gravity.

Can we get a bounce instead of a singularity?