Geons in Asymptotically Anti-de Sitter spacetimes

Grégoire Martinon

Observatoire de Paris
Université Paris Diderot

6 Juillet 2015

AdS/CFT correspondance

(Anti-de Sitter/Conformal Field Theory)

AdS/CFT

Seminal paper

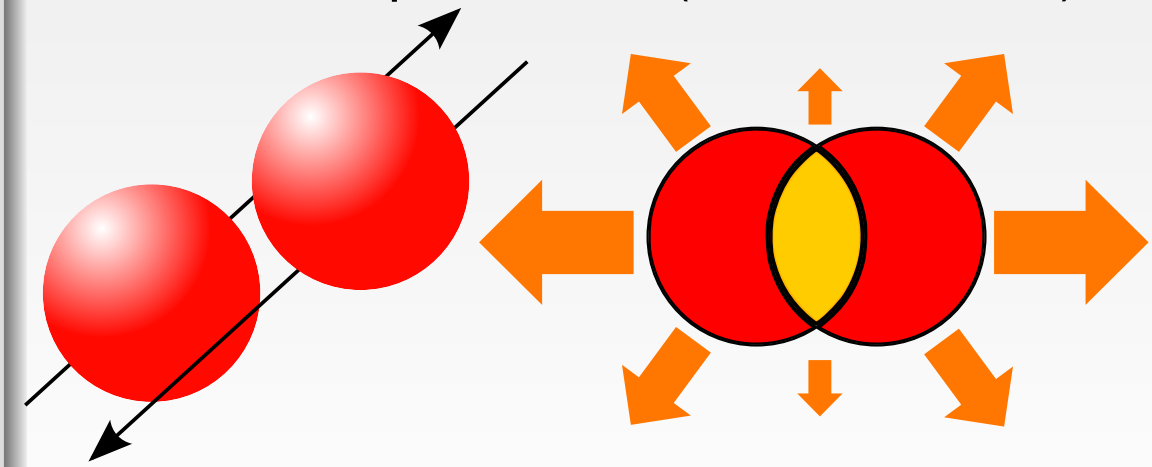
The Large N Limit of Superconformal Field Theories and Supergravity
J. M. Maldacena, *Adv. Theor. Math. Phys.* **2** (1998) 231

More than 7400 citations !

Applications

- QCD
- nuclear physics
- nonequilibrium physics
- condensed-matter physics
- **Quark-gluon plasma**

AdS/CFT experiments (RHIC, LHC...)



Holographic principle

Strongly coupled 4D gauge theory = Gravitational theory in 5D AAdS

- QCD, QED at strong coupling is hard
- Super gravity in AAdS much easier
- AdS_5 dual to $\mathcal{N} = 4$ Super Yang-Mills

Black hole thermodynamics

- Because of Hawking radiation :

  AAdS without Black Hole : $T = 0$
  AAdS with Black Hole : $T \neq 0$

- BH thermodynamics : $S \propto A$
  5D area $\simeq$ 4D volume

Anti-de Sitter (AdS) spacetime

Negative cosmological constant

AdS = the maximally symmetric solution of Einstein equation with $\Lambda < 0$

$$R_{\mu\nu} - \frac{R}{2} g_{\mu\nu} + \Lambda g_{\mu\nu} = 0$$

Constant curvature : $\boxed{R = -\frac{12}{L^2}}$

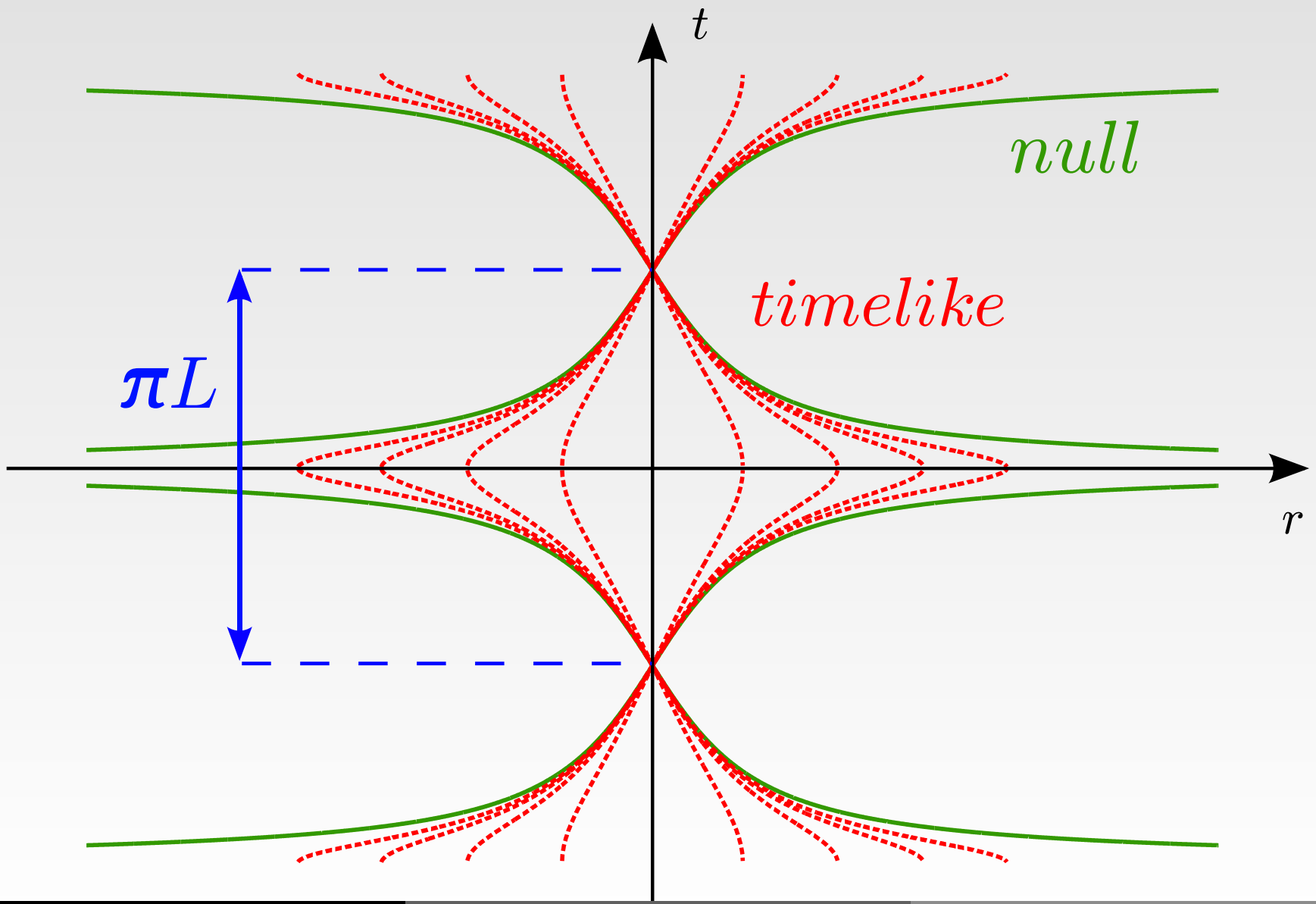
10 Killing vectors that are (in the $L \to +\infty$ limit) :

- 4 spacetime translations
- 3 spatial rotations
- 3 Lorentz boosts

AdS size

- AdS length L : $\boxed{\Lambda = -\frac{3}{L^2}}$
- $\lim_{L \to +\infty}$ AdS = Minkowski

Radial geodesics



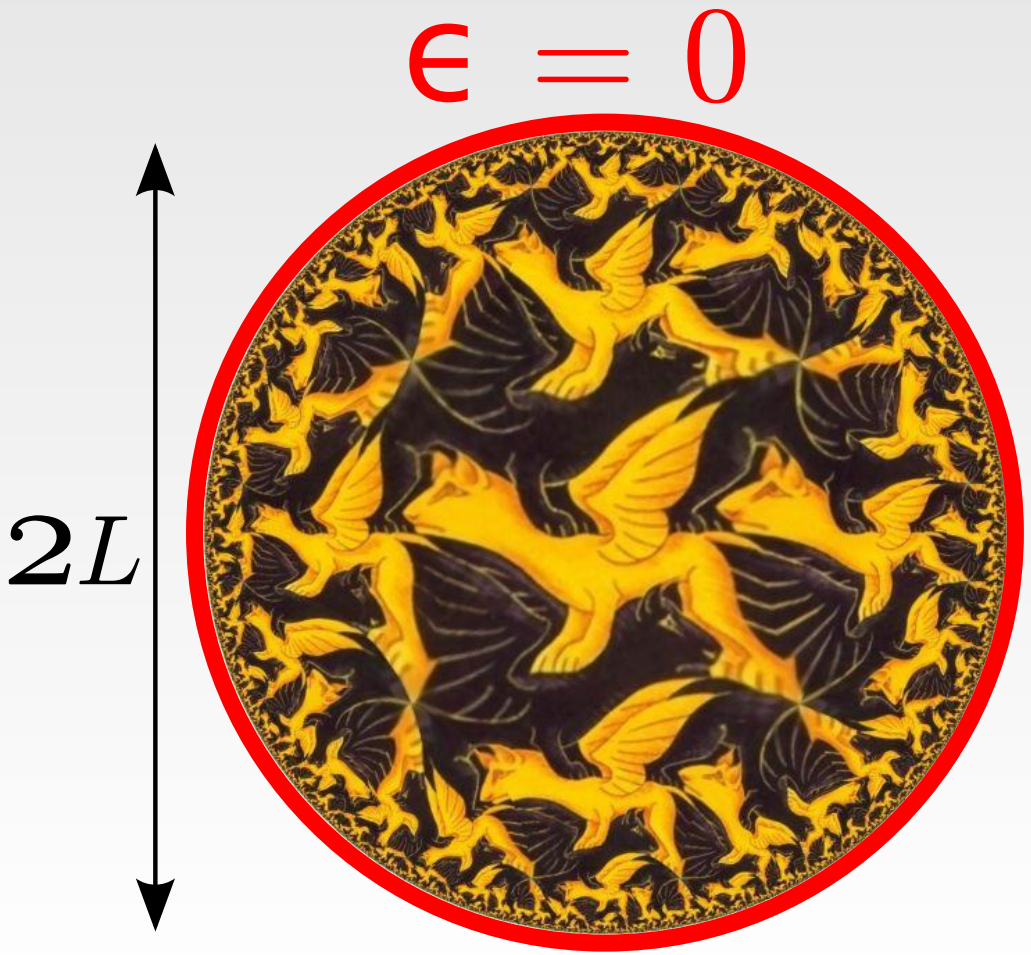
Conformal representation

$$ds^2 = -\left(\frac{1+\rho^2}{1-\rho^2}\right)^2 dt^2 + \frac{4}{(1-\rho^2)^2}\left[dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2)\right] \qquad \left(\rho = \frac{r}{L}\right)$$

- 3-metric conformally flat
- compactified $r \in [0, L]$
- boundary at $\varepsilon \equiv 1 - \frac{r}{L} = 0$

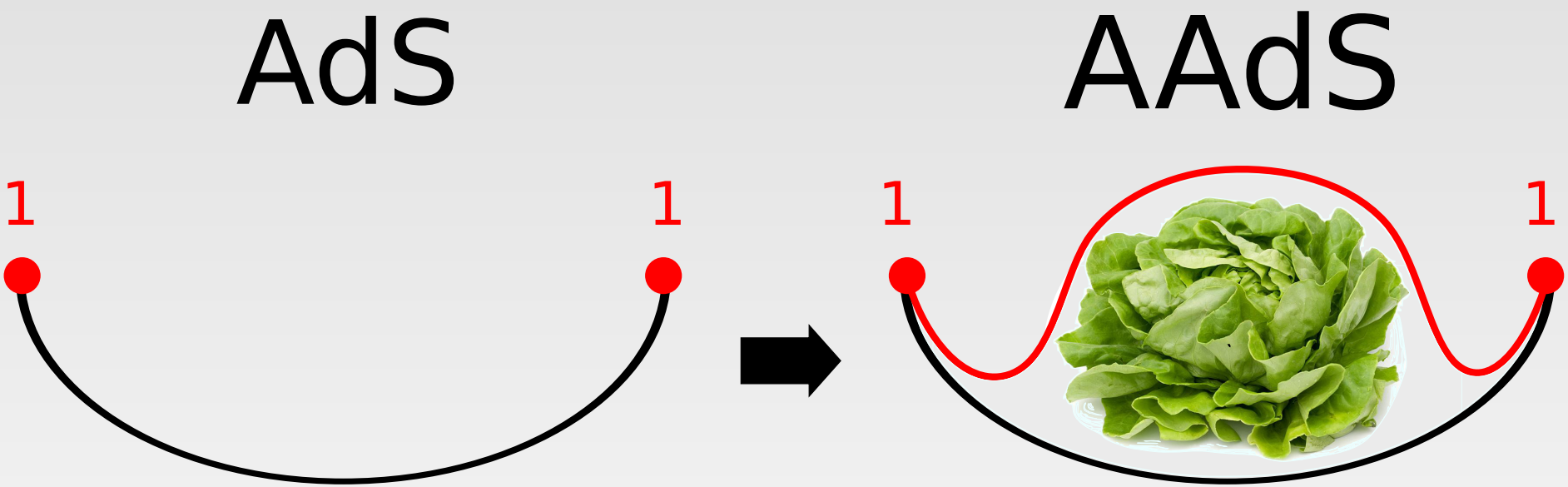
Asymptotic behaviour

- $\bar{N} = O\left(\frac{1}{\varepsilon}\right)$
- $\bar{\gamma}_{ij} = O\left(\frac{1}{\varepsilon^2}\right)$; $\bar{\gamma}^{ij} = O\left(\varepsilon^2\right)$
- $\bar{\Gamma}^k_{ij} = O\left(\frac{1}{\varepsilon}\right)$
- $\bar{R}_{ij} = O\left(\frac{1}{\varepsilon^2}\right)$



Asymptotically AdS (AAdS) spacetime

AdS + something



Problem : $\bar{\gamma}_{ij}$ diverges like $O\left(\frac{1}{\varepsilon^2}\right)$ at the boundary !
Cure : conformal structure at the boundary

AAdS boundary conditions

$$\varepsilon N \underset{r=L}{=} \varepsilon \bar{N} = 1 \quad ; \quad \beta^i \underset{r=L}{=} \bar{\beta}^i = 0 \quad ; \quad \varepsilon^2 \gamma_{ij} \underset{r=L}{=} \varepsilon^2 \bar{\gamma}_{ij} = 1$$

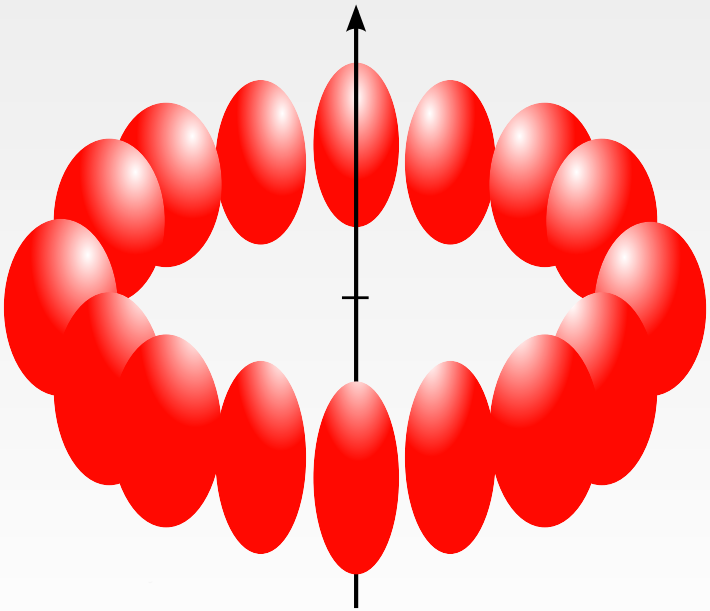
Geons in AAdS spacetime

What's a geon ?

GEON = Electro-Gravitational Entity

Seminal papers : 50's and 60's

Wheeler, Power, Brill, Ernst,
Melvin, Hartle, Thorne, Kaup



At the beginning :

- asymptotically flat
- EM/ν/GW/ϕ wave packet
- cylindrical, toroidal, spherical
- variational principle
- intricate geodesics

Geon properties

- need rotation to avoid collapse
- photon self attraction
- Black body radiation

Linear geon in AAdS

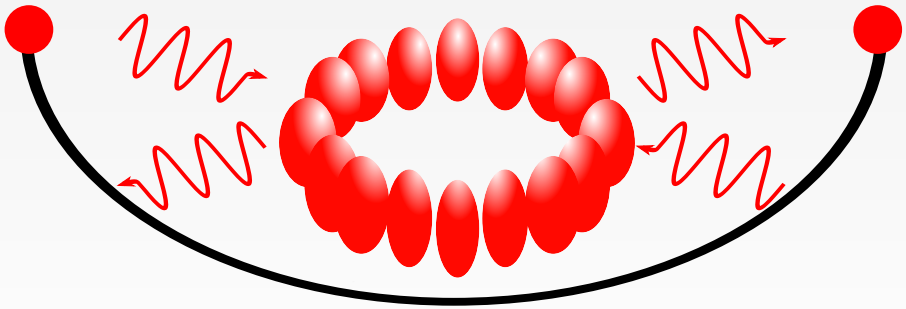
Seminal papers (AAdS) : 2010's

Bizon, Rostworowski, Maliborski, Dias, Horowitz, Santos, Kodama, Ishibashi, Seto, Wald

Properties

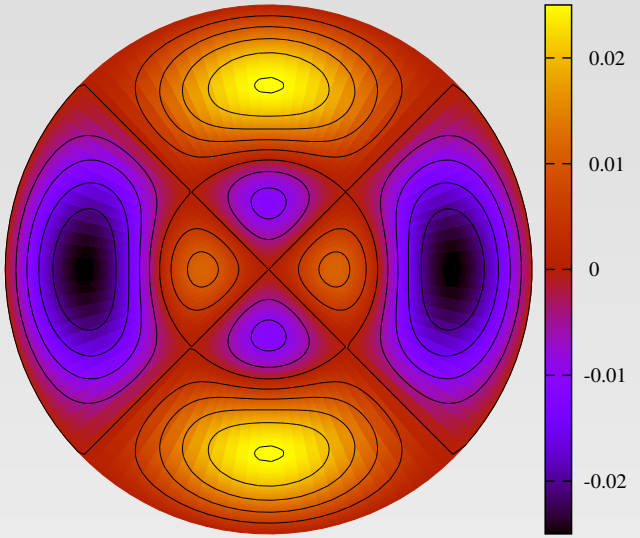
- vaccum solution
- stable GW wave packet
- need rotation to avoid collapse
- stationnary in corotating frame

$\Rightarrow$ Helical Killing vector

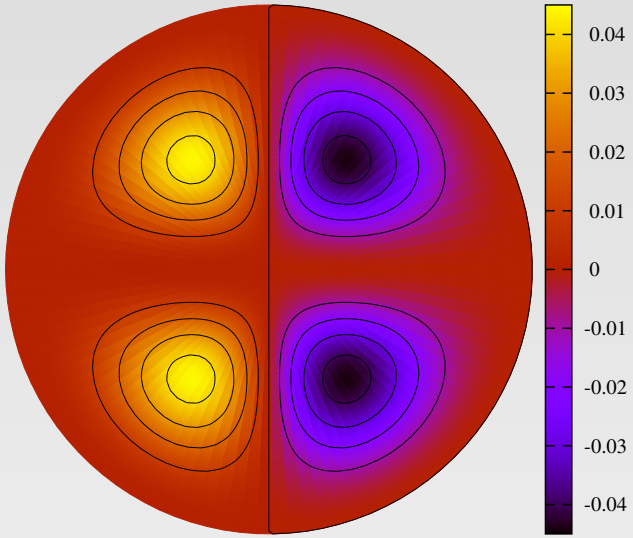


Mathematics

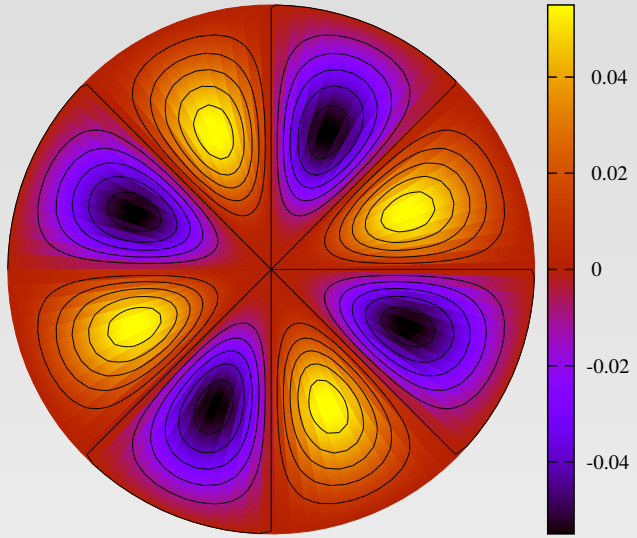
- $\gamma_{ij} = \bar{\gamma}_{ij} + \mathcal{A} h_{ij}$ with $\mathcal{A} \ll 1$
- $R_{\mu\nu} - \frac{R}{2} g_{\mu\nu} + \Lambda g_{\mu\nu} = O(\mathcal{A}^2)$
- Tensor spherical harmonics



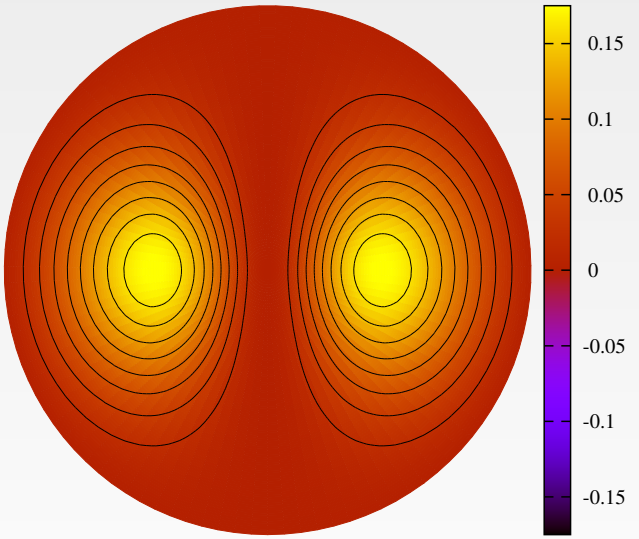
$\hat{n}$ in z = 0 plane



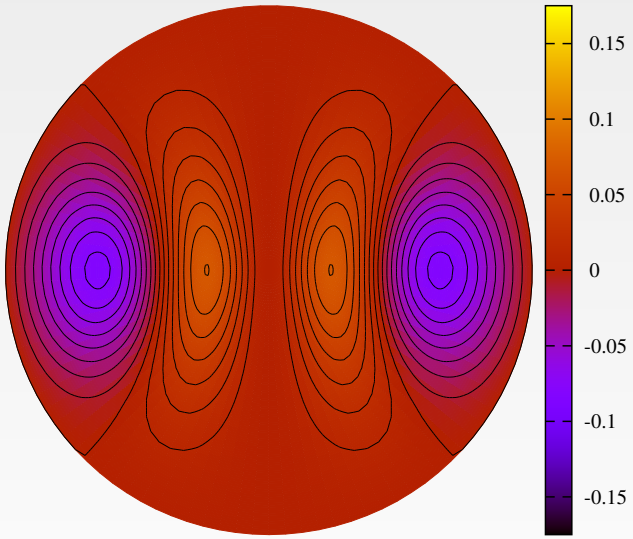
B^y in z = 0 plane



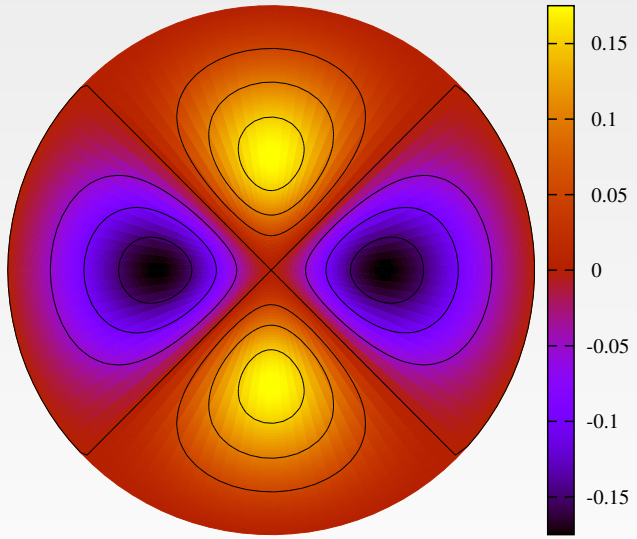
$\hat{h}_{xy}$ in z = 0 plane



$\hat{h}_{xx}$ in x = 0 plane



$\hat{h}_{yy}$ in x = 0 plane



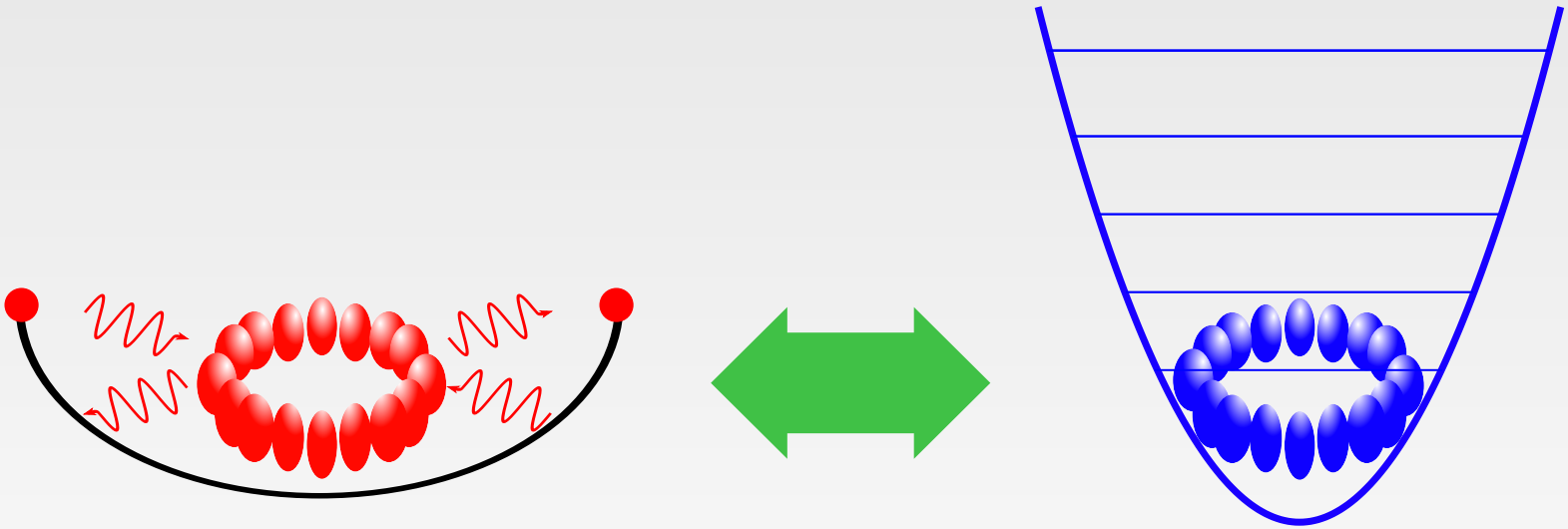
$\hat{h}_{zz}$ in z = 0 plane

What is a geon useful for ?

- AdS/CFT dual representation : No black hole $\Rightarrow$ $\boxed{T = 0}$

Geon in AAdS

Spin-2 Bose-Einstein



- Explore a non-linear stability island of AdS spacetime [1, 2]

---

1. P. Bizoń and A. Rostworowski. *Physical Review Letters*, 107, July 2011
2. P. Bizoń, M. Maliborski, and A. Rostworowski. *ArXiv*, June 2015, 1506.03519

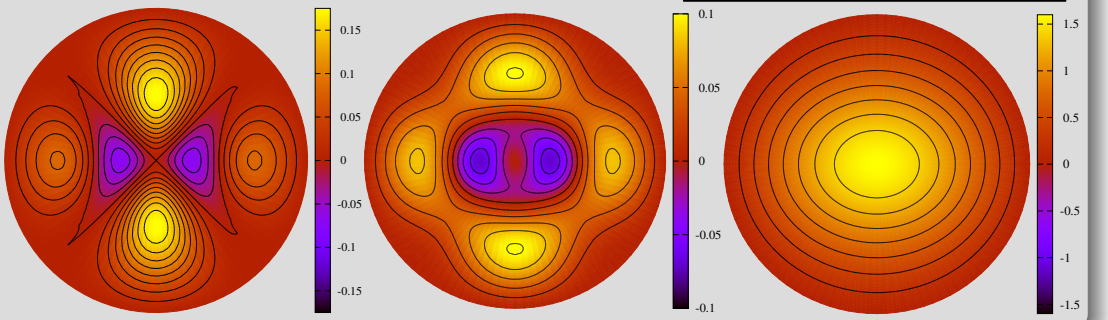
Non-linear geon

Step by step

1. Find linear geon
2. AAdS-AdS formulation
3. Choose a gauge
4. Build good initial guess
5. Invert Einstein system

Gauge choices

$\hat{h}_{xx}$ $(z = 0)$ | $K = 0$ | $\gamma^{kl}(\Gamma^i_{kl} - \bar{\Gamma}^i_{kl}) = 0$



What you need :

- Kodama-Ishibashi formalism
- Spectral method
- Numerical library : KADATH
- Newton-Raphson in dimension 10^5
- Many computers ! (at least 200)

Conclusion

Conclusion

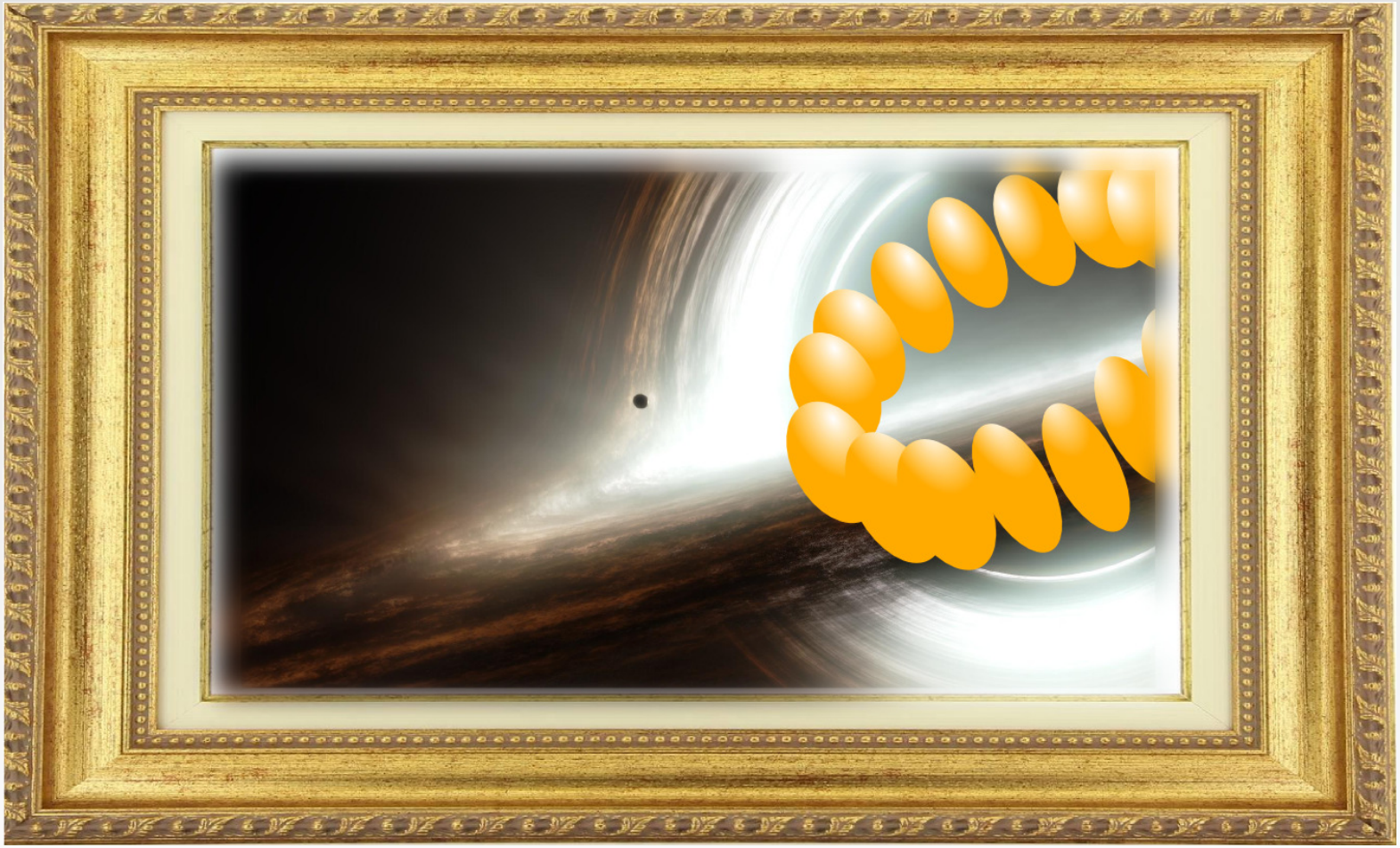
State of the art :

- first geon obtained at medium resolution
- ready for sequence computation

TO-DO list :

- ask hours on regional and national supercomputers
- compute geon sequences
- compare global quantities (M, J) with linear solution
- maximum mass and maximum angular momentum of a geon ?
- check stability with evolution code
- see if the geon admits a limit in the $\Lambda \to 0$ limit
- add ingredients (black holes, boson stars...)

Thank you for your attention



Artist's view of a geon

References


P. Bizoń and A. Rostworowski.
Physical Review Letters, 107, July 2011.


P. Bizoń, M. Maliborski, and A. Rostworowski.
ArXiv, June 2015, 1506.03519.


L. Andersson and V. Moncrief.
Elliptic-hyperbolic systems and the einstein equations.
Annales Henri Poincaré, 4(1) :1–34, 2003.


Dennis M. DeTurck.
Deforming metrics in the direction of their ricci tensors.
J. Differential Geom., 18(1) :157–162, 1983.

References (cont.)

M. Headrick, S. Kitchen, and T. Wiseman.
A new approach to static numerical relativity and its application to Kaluza-Klein black holes.
Classical and Quantum Gravity, 27(3) :035002, February 2010, 0905.1822.

P. Figueras, J. Lucietti, and T. Wiseman.
Ricci solitons, Ricci flow and strongly coupled CFT in the Schwarzschild Unruh or Boulware vacua.
Classical and Quantum Gravity, 28(21) :215018, November 2011, 1104.4489.

Juan Maldacena.
The large-n limit of superconformal field theories and supergravity.
International Journal of Theoretical Physics, 38(4) :1113–1133, 1999.

References (cont.)

H. Kodama, A. Ishibashi, and O. Seto.
Phys. Rev. D, 62(6) :064022, September 2000, hep-th/0004160.

A. Ishibashi and R. M. Wald.
Classical and Quantum Gravity, 21 :2981–3013, June 2004, hep-th/0402184.