

# Journée GPhys 2015

Institut d'Astrophysique de Paris

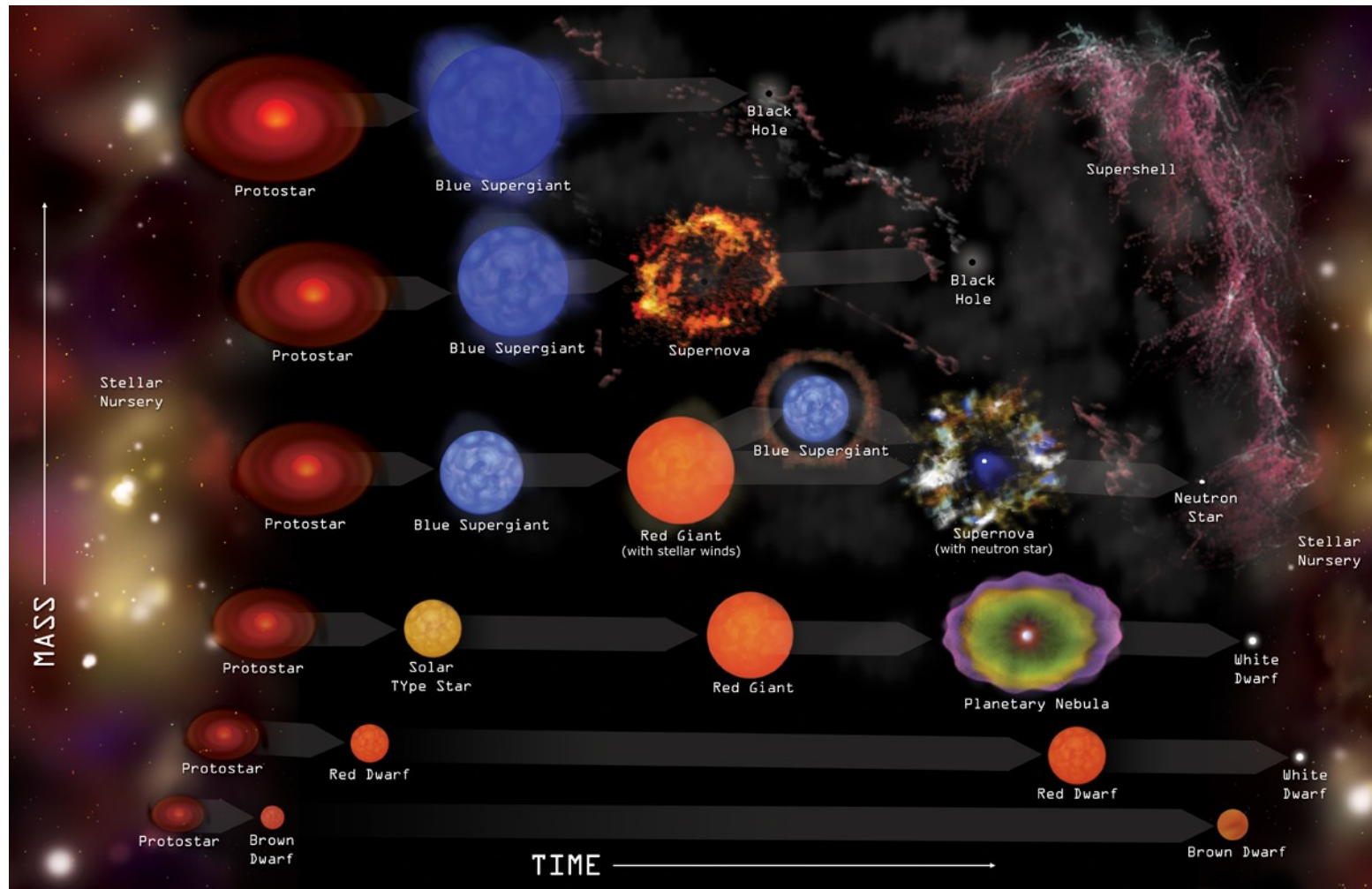
## Relativistic Rotating Stars With Hot Matter

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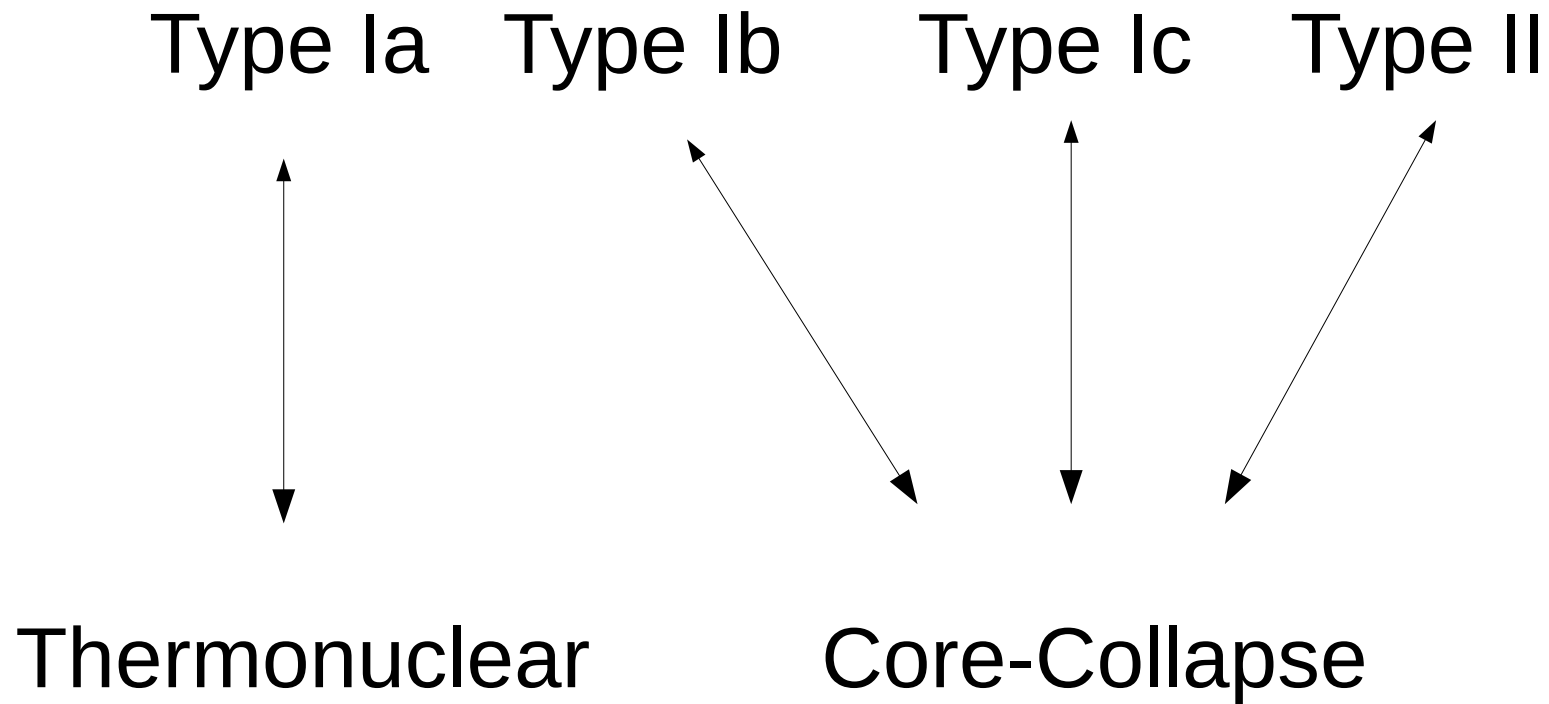
# How do Massive Stars Die?

- The final fate of star evolution



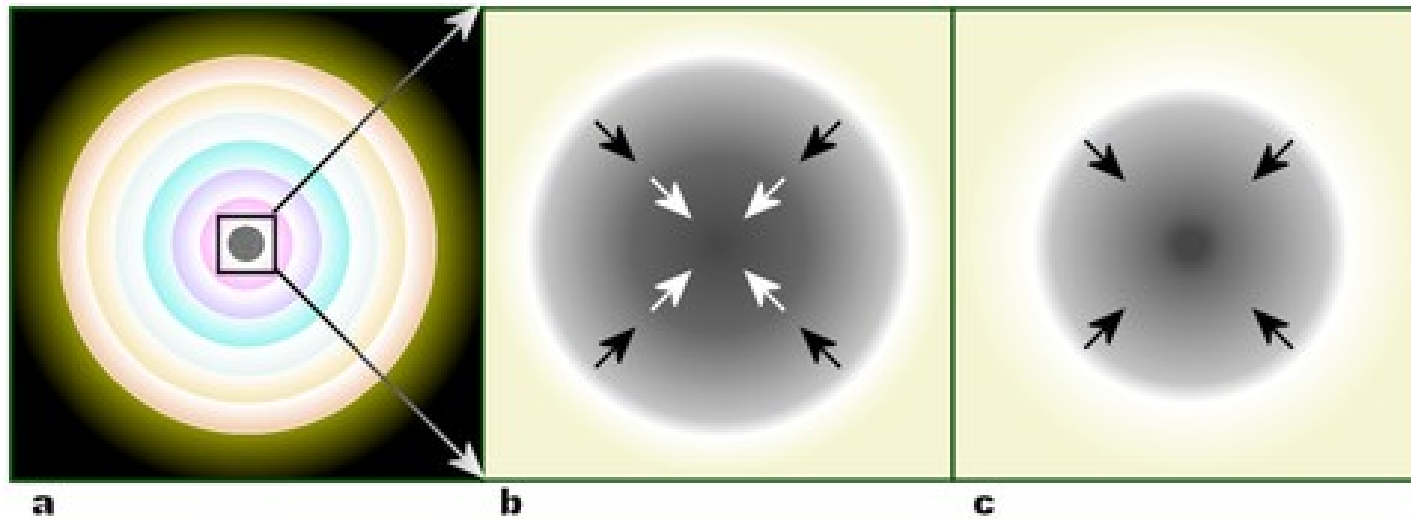
# How do Massive Stars Die?

- Which supernovae?



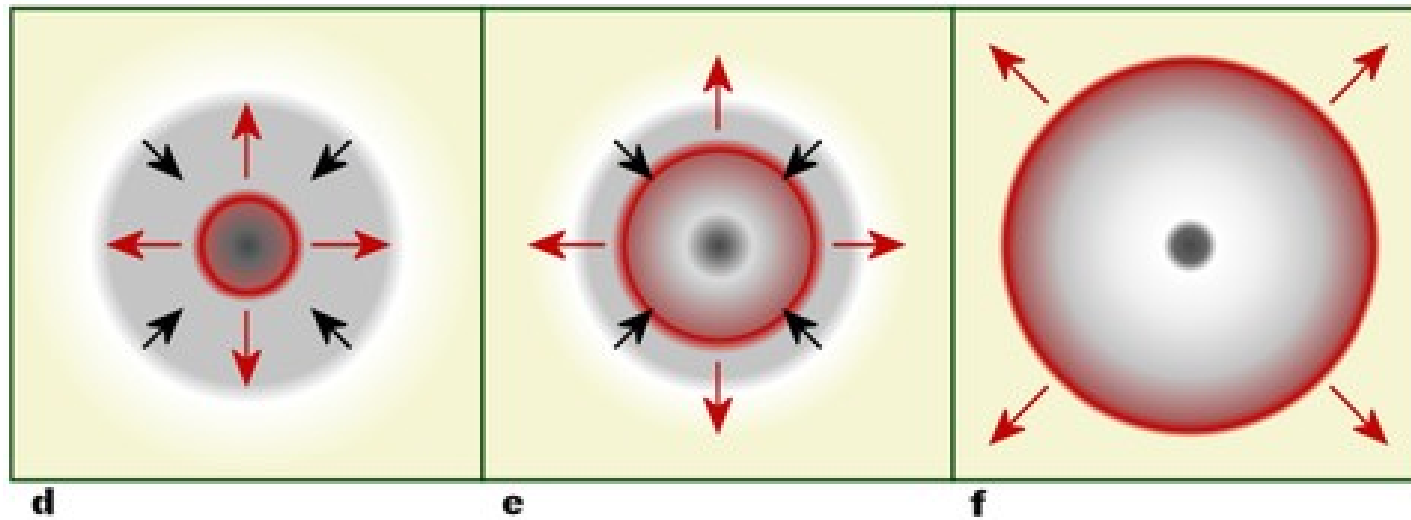
# Core-Collapse

- The gravitational collapse of the iron core



# Core-Collapse

- The consequences of the core-collapse

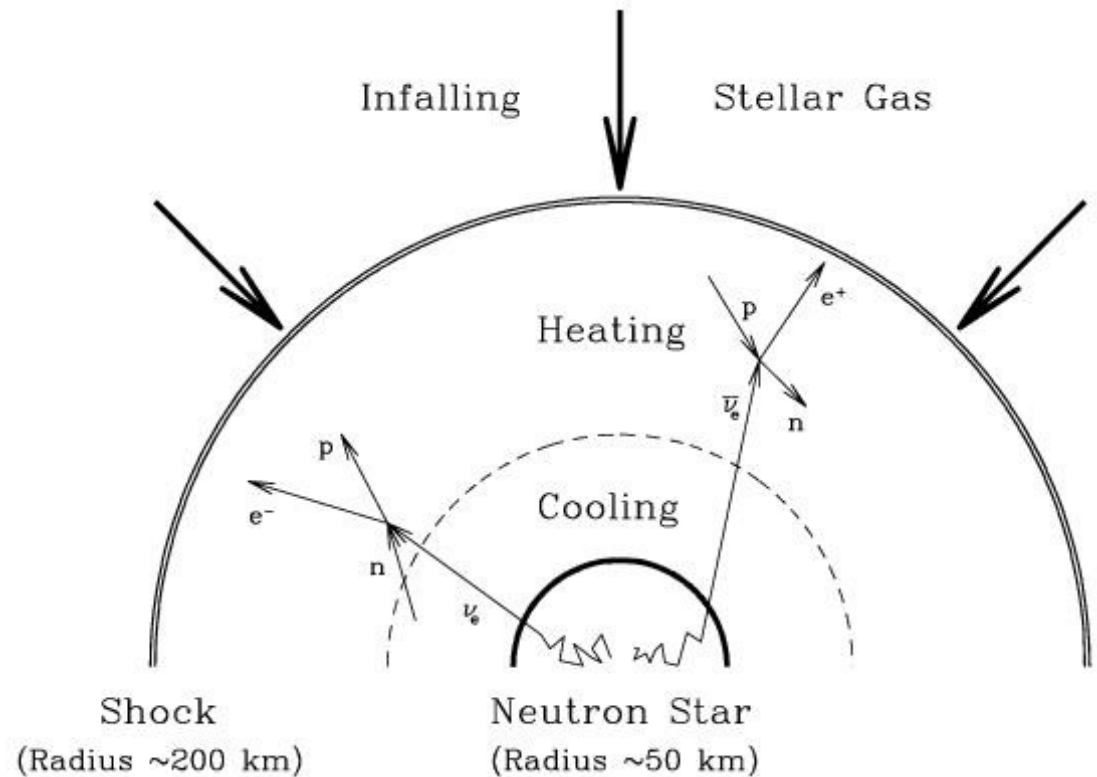


# Core Collapse

- The neutrino driven mechanism

## Explosion mechanisms

- The neutrino heating
- Hydrodynamical instabilities (Sasi, convection)
- Magnetorotational instabilities
- ...



# Finite Temperature in Compact Stars

- Core-Collapse Supernovae

-  $T \sim 50 - 100 \text{ MeV}$

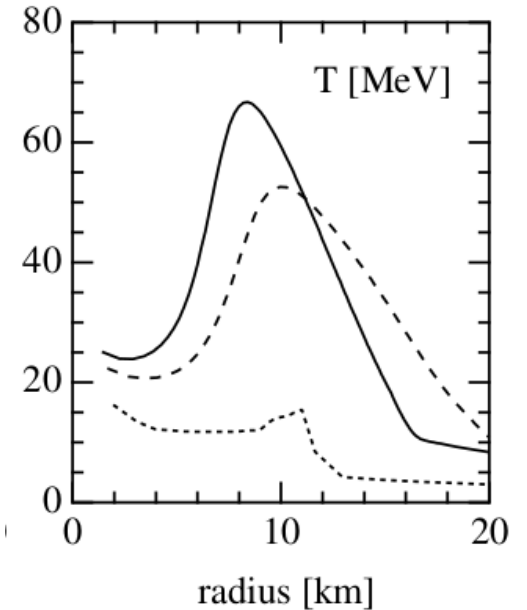
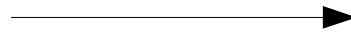


- Neutron Star Mergers

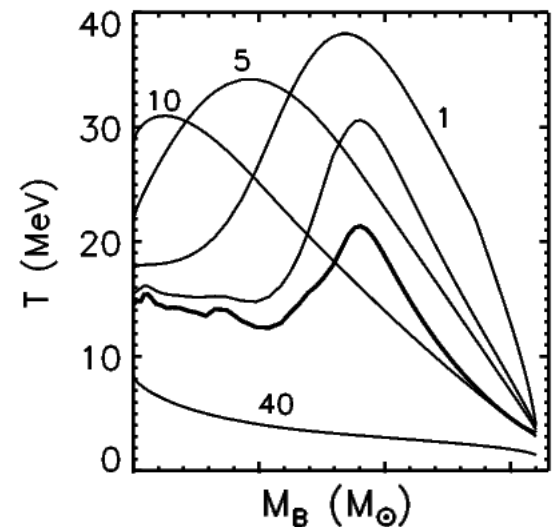
-  $T \sim 80 \text{ MeV}$

- Proto-Neutron Stars

-  $T \sim 50 \text{ MeV}$



Fischer (2009)



Ferrari et al (2003)

$$10 \text{ MeV} \approx 1.16 \times 10^{11} \text{ K}$$

# Finite Temperature in Compact Stars

- Core-Collapse Supernovae
  - Temperature effects included since many years in dynamical simulations
- Neutron Star Mergers
  - Kaplan et al (2014), Bauswein et al (2012), Abdikamalov et al (2013), Sekiguchi et al (2011) ...
- Proto-Neutron Stars
  - Martinon et al (2014), Villain et al (2004), Pons et al (1999), Goussard et al (1998, 1997) ...



# Finite Temperature in Compact Stars

- Finding a general solution for the equilibrium equations of stationary axisymmetric spacetimes with finite temperature is difficult. Typical strategies are:

- Effective barotropic EoS

Goussard et al (97,98), ...

$$p = p(n_b, T(n_b), Y_e(n_b, T(n_b)))$$

- Using perturbative approximation methods for the metric

Martinon et al (2014)

# Our model for hot stars

- General relativistic, stationary axisymmetric solutions of rotating stars

- Perfect fluid

Energy-momentum tensor  $\rightarrow T^{\mu\nu} = (p + \varepsilon)u^\mu u^\nu + p g^{\mu\nu}$

- Temperature (or entropy) dependent EoS's

$$p := p(n_b, s_b)$$

$$\varepsilon := \varepsilon(n_b, s_b)$$

# Spacetime Solution

- Stationary axisymmetric spacetimes

Two killing vector fields:

$$\partial_t, \partial_\phi$$

We use a conformal 3-metric, defined as:

$$\tilde{\gamma}_{ij} := \Psi^{-4} \gamma_{ij}$$

- Dirac Gauge

(Bonazzola et al, 2004)

(Lin and Novak, 2006)

$$\mathcal{D}_j h^{ij} = 0,$$

$$h^{ij} := \tilde{\gamma}^{ij} - f^{ij}$$

- Maximal slicing condition

$$K = 0$$

# Star equilibrium

- Equilibrium equations (for rigid rotation)

$$\nabla_{\mu} T^{\mu\nu} = 0$$

Defining the pseudo-log enthalpy field as

$$H = \ln \left( \frac{\varepsilon + p}{m_b n_b} \right),$$

With the help of the 1<sup>st</sup> law of thermodynamics, the equilibrium equations read

$$\partial_i (H + \nu - \ln \Gamma) = T e^{-H} \partial_i s_b, \quad i = r, \theta \quad \text{Goussard et al, 97}$$

# Star equilibrium

- Equilibrium equations (for rigid rotation)

$$\partial_i (H + \nu - \ln \Gamma) = T e^{-H} \partial_i s_b, \quad i = r, \theta$$

No first integral in general! First integral for the barotropic EoS  $H(n_b)$ .

# Star equilibrium

- Equilibrium equations (for rigid rotation)

$$\partial_i (H + \nu - \ln \Gamma) = T e^{-H} \partial_i s_b, \quad i = r, \theta$$

Instead of solving an analytical first integral, we propose the following scheme

$$H = -\nu + \ln \Gamma + \int_0^{r_s} T e^{-H} \partial_r s_b dr'$$
$$\Delta_{\theta\phi} s_b = \left( \partial^\theta + \frac{1}{\tan \theta} \right) \frac{\partial_\theta (T e^{-H})}{\partial_r (T e^{-H})} \partial_r s_b$$

Where the monopolar part of the  $s_b$  has to be specified, whereas the higher multipoles are determined by the equilibrium solver

# Results

- To test the code, we use the relativistic ideal gas EoS:

$$p(H, s_b) = k n_b(H, s_b)^\gamma e^{(\gamma-1)s_b}$$

$$\varepsilon(H, s_b) = \frac{k}{\gamma - 1} n_b(H, s_b)^\gamma e^{(\gamma-1)s_b} + m_b n_b(H, s_b)$$

$$T(H) = m_b \frac{\gamma - 1}{\gamma k_b} (e^H - 1)$$

$$n_b(H, s_b) = \left( m_b \frac{\gamma - 1}{\gamma k} (e^H - 1) \right)^{\frac{1}{\gamma-1}} e^{-s_b}$$

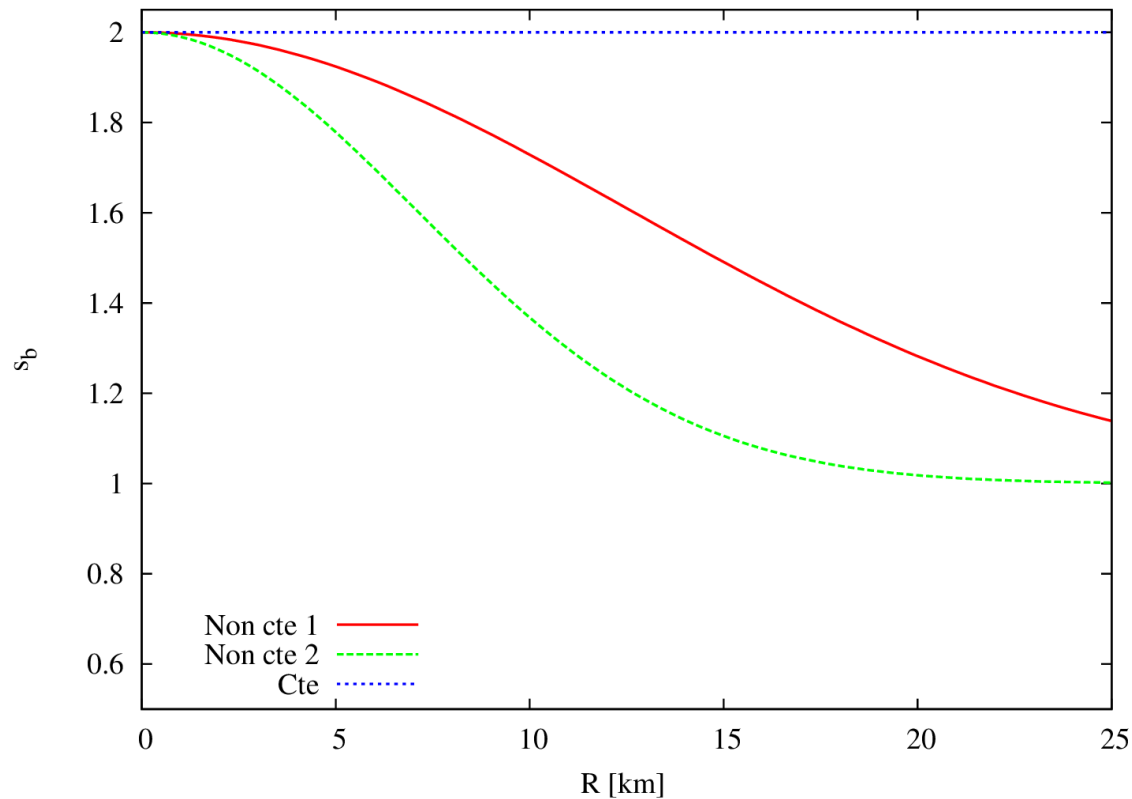
with  $\gamma = 2$  and  $k = 0.026 \rho_{\text{nuc}} c^2 / n_{\text{nuc}}$

# Results

- We will consider three different entropy per baryon profiles

$$\text{non cte 1} \rightarrow s_b = 1 + e^{-\frac{r^2}{10^{2.5}}}$$

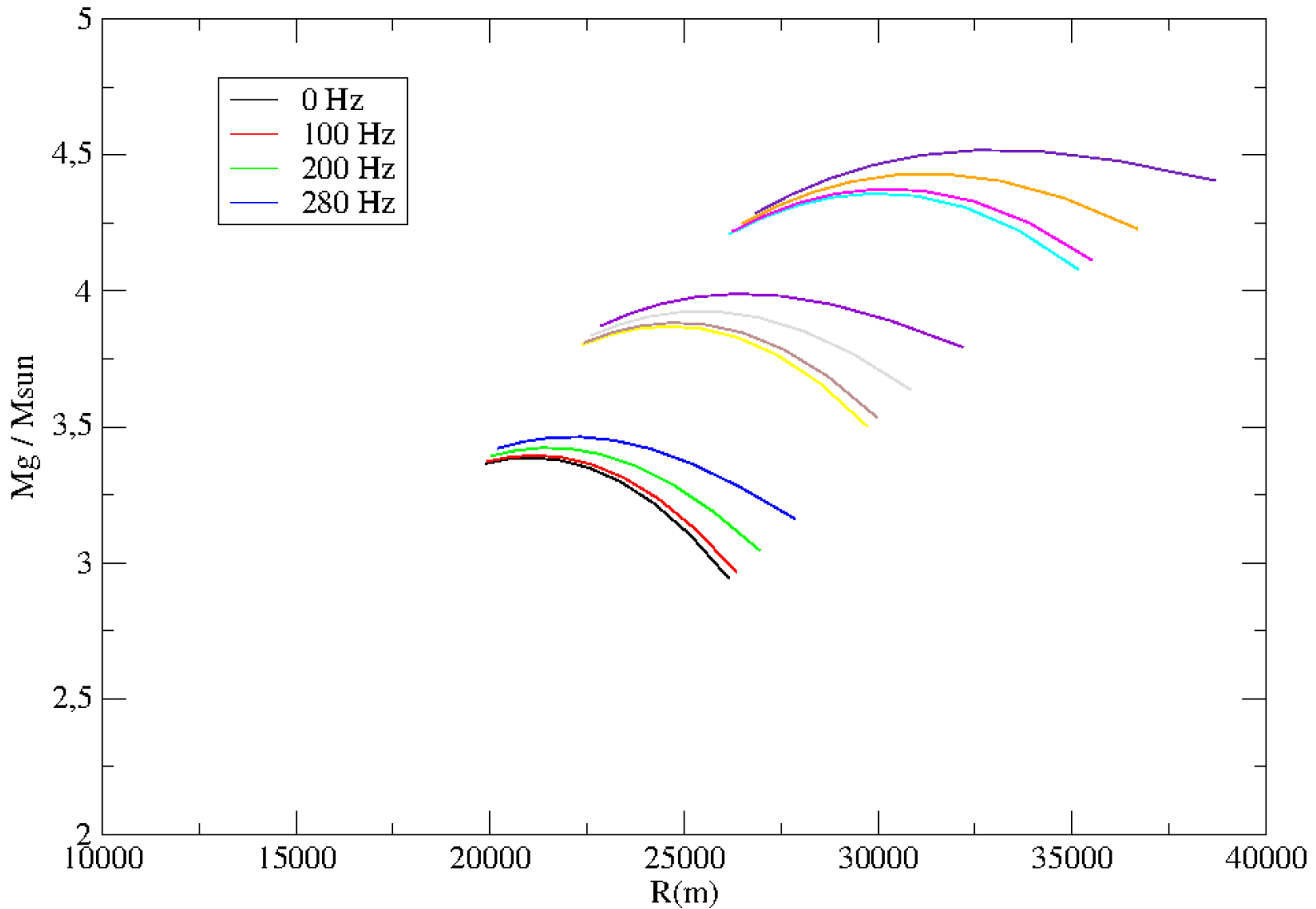
$$\text{non cte 2} \rightarrow s_b = 1 + e^{-\frac{r^2}{10^2}}$$





# Results

- Mass/Radius profiles



# Results

Constant sb profile

Rot = 373.5 Hz

Central H =  $0.3 c^2$

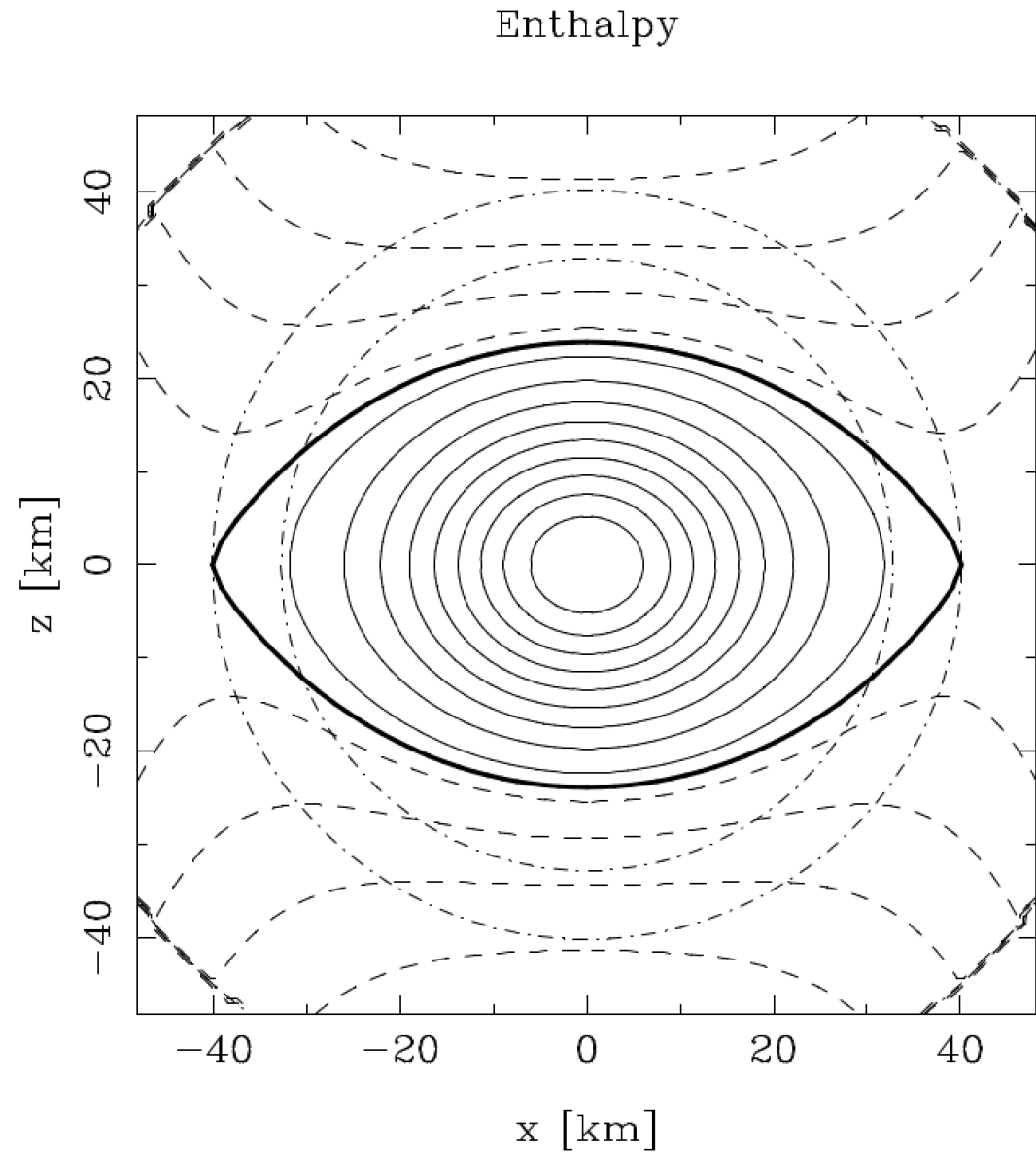
Equatorial r = 60.21 Km

Gravitational M =  $4.83 M_{\text{sol}}$

Rpole/Req = 0.39

GRV2 =  $-8.84 e-6$

GRV3 =  $-1.04 e-5$



# Results

Non constant sb profile 2

Rot = 310 Hz

Central H =  $0.3 c^2$

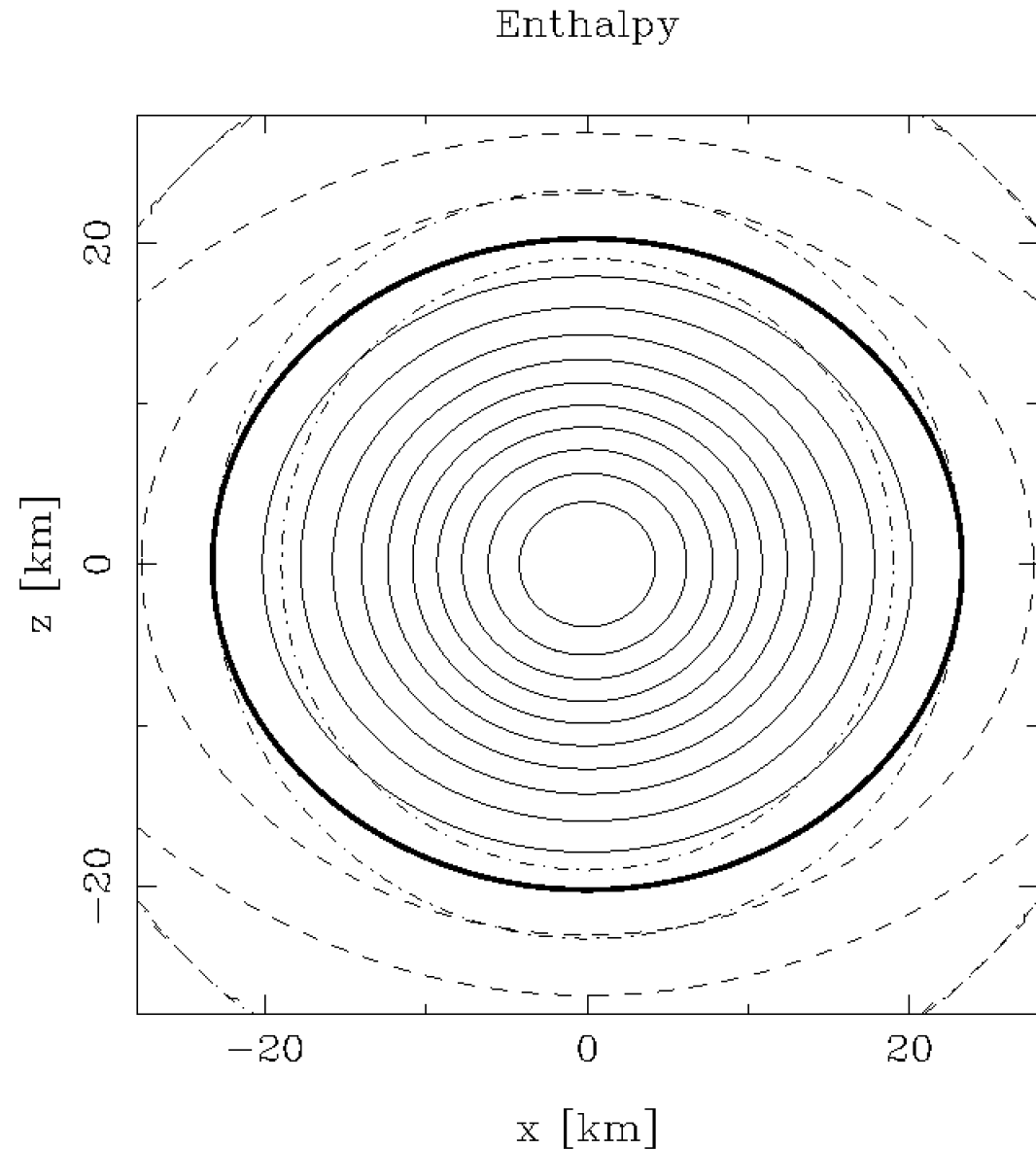
Equatorial r = 32.19.21 Km

Gravitational M =  $3.79 M_{\text{sol}}$

Rpole/Req = 0.86

GRV2 =  $3.12 e-6$

GRV3 =  $5.94 e-7$



# Conclusions and future perspectives

- We propose a numerical scheme to consistently make use of not necessarily barotropic EoS capable of modeling appropriately the finite temperature effects on a stationary axisymmetric star, using general entropy profiles
- Having in mind the use of realistic EoS's, we intend to extend the code for EoS's with electron fraction as a third parameter
- We plan to use this for developing quasi-stationary models of proto-neutron stars

**Thank You!**