Celestial mechanics in Boson Star spacetime

New type of orbits

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Outline

New observations with GRAVITY

GRAVITY instrument

- Optical interferometry in the near-infrared
- astrometric precision of 10 μas on each orbit
- Possibility to observe stellar orbits near the Galactic center

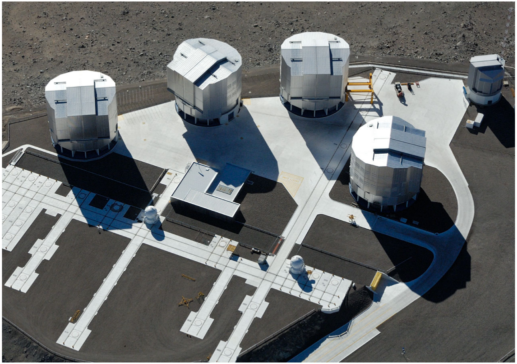


Figure: Four 8 m telescopes at VLT (Chile)

Sgr A* : Kerr Black Hole versus Boson Star

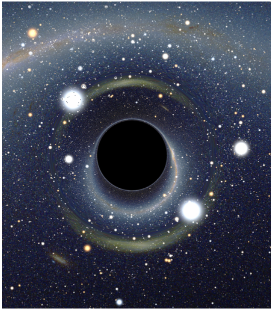


Figure: Image of a Schwarzschild Black Hole

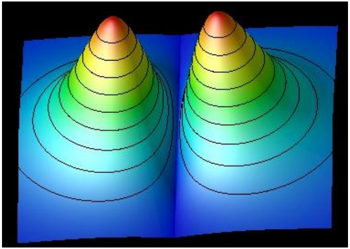


Figure: Rotating Boson Star

Idea : compare the timelike geodesics in those two spacetimes

Field equations

Boson Star : gravitationally bound state of a complex scalar field

ϕ which is solution of the following system

- Einstein equations

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi T_{\mu\nu}$$

$$T_{\mu\nu} = \frac{1}{2}\left[\nabla_\mu \bar{\phi} \nabla_\nu \phi + \nabla_\mu \phi \nabla_\nu \bar{\phi}\right] - \frac{1}{2} g_{\mu\nu}\left[g^{\gamma\delta} \nabla_\gamma \bar{\phi} \nabla_\delta \phi + V\left(\mid \phi \mid^2\right)\right]$$

- Klein Gordon equation

$$\nabla_\mu \nabla^\mu \phi = \frac{\mathrm{d} V}{\mathrm{d} \mid \phi \mid^2} \phi$$

Here we consider “mini” boson stars with $V\left(\mid \phi \mid^2\right) = \frac{m^2}{\hbar^2} \mid \phi \mid^2$

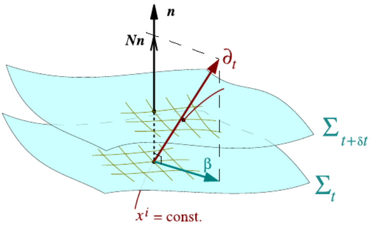
Stationary and axisymmetric solution

Assumptions

- Stationarity and axisymmetry for the spacetime metric $g_{\alpha\beta}$
- Ansatz for the field ϕ

$$\phi = \phi_0\,(r,\theta)\,e^{i(\omega t - k\varphi)}$$

with $\phi_0\,(r,\theta)$ a real function, $\omega \in \mathbb{R}$ and k is an integer.



Solutions found by Kadath using the 3+1 formalism

$$g_{\alpha\beta}dx^{\alpha}dx^{\beta} = -N^2\mathrm{d}t^2 + A^2\left(\mathrm{d}r^2 + r^2\mathrm{d}\theta^2\right) + B^2 r^2 \sin^2\theta\,(\mathrm{d}\varphi + \beta^{\varphi}\mathrm{d}t)^2$$

Plots of the boson star field

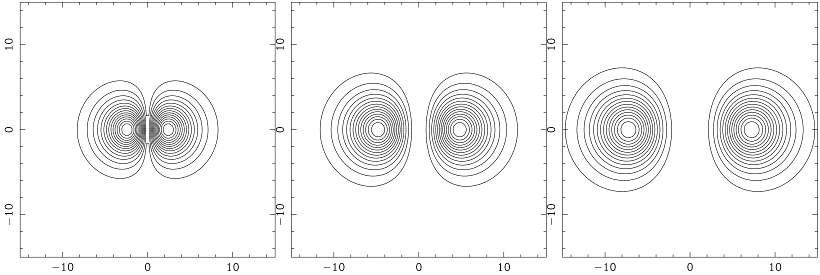


Figure: Isocontours of $\phi_0(r,\theta)$ in the plane $\varphi = 0$ for $\omega = 0.8\ m/\hbar$:

$$\phi = \phi_0(r,\theta)e^{i(\omega t - k\varphi)}$$

with $k = 1$; $k = 2$; $k = 3$

Could Sgr A* be a Higgs Star ?

Mass of the Higgs boson

$$m_H = 125.3 \pm 0.6\,\mathrm{GeV}$$

- Mini-boson star $V(|\phi|^2) = \frac{m^2}{\hbar^2} |\phi|^2$

$$M_{crit} = 3.10^9 \mathrm{kg} \ll M_{SgrA*} = 9.10^{36}\ \mathrm{kg}$$

- Boson star $V(|\phi|^2) = \frac{m^2}{\hbar^2} |\phi|^2 \left(1 + 2\pi\Lambda |\phi|^2\right)$ with $\Lambda = 200$:

$$M_{crit} = 8.10^{26} \mathrm{kg} \ll M_{SgrA*}$$

- Solitonic boson star $V(|\phi|^2) = \frac{m^2}{\hbar^2} |\phi|^2 \left(1 - \frac{|\phi|^2}{\sigma^2}\right)^2$ with $\sigma = m_H$:

$$M_{crit} = 4.10^{41} \mathrm{kg} \sim M_{SgrA*}$$

Effective potential

Equation for r : $\left(\frac{dr}{d\tau}\right)^2 = \mathcal{V}_{eff}\left(r, \epsilon, \ell\right)$

with $\mathcal{V}_{eff}\left(r, \epsilon, \ell\right) = \frac{1}{A^2}\left[\frac{1}{N^2}\left(\epsilon + \beta^{\varphi}\ell\right)^2 - \frac{\ell^2}{B^2 r^2} - 1\right]$

$$\mathcal{V}_{eff} \geq 0 \Rightarrow \epsilon \geq \epsilon_{min}$$

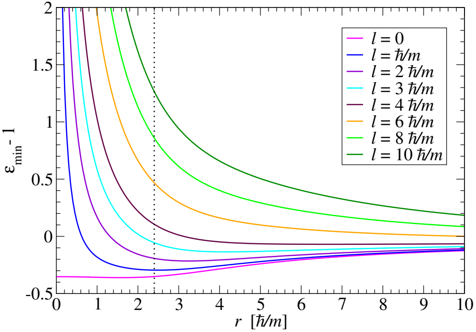


Figure: Effective potential profiles for $\omega = 0.8\ m/\hbar$ and $k = 1$

Zero angular momentum orbits

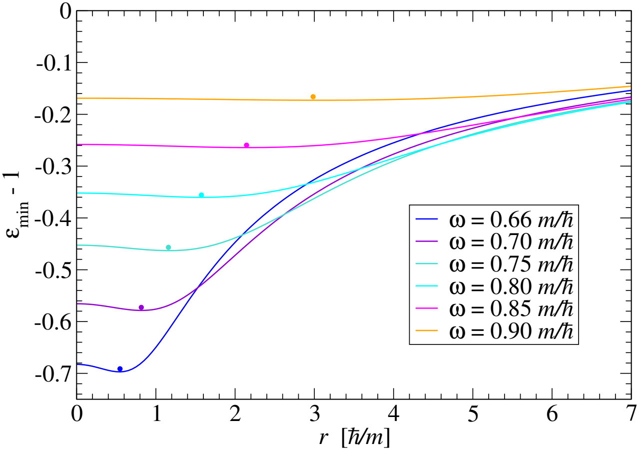


Figure: Effective potential for $\ell = 0$ for $k = 1$ and different values of ω

Pointy Petal orbits

Using GYOTO ray-tracing code :

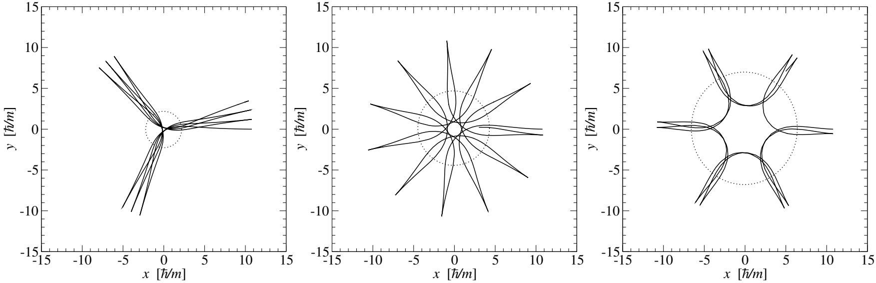


Figure: Orbit of a $\ell = 0$ test particle in the equatorial plane of a boson star with $\omega = 0.8\ m/\hbar$ and $k = 1, 2, 3$

Pointy Petal orbits

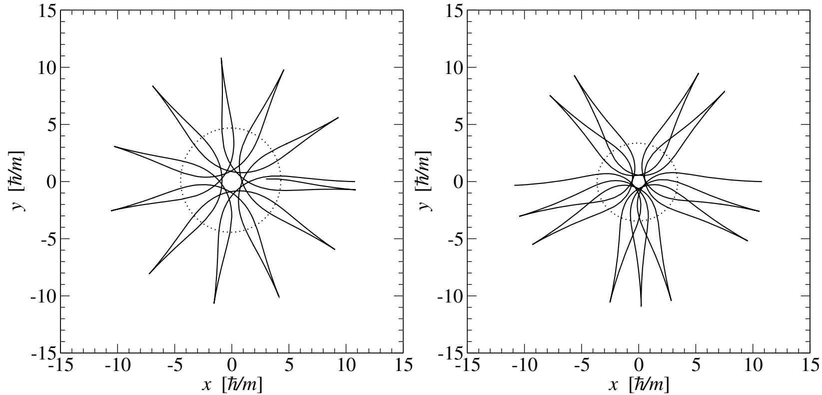


Figure: Orbit of a $\ell = 0$ test particle in the equatorial plane of a boson star with $k = 2$ and $\omega = 0.8\ m/\hbar$ and $0.75\ m/\hbar$

Conclusion

New type of orbits with pointy petals

- In Kerr spacetime, orbits with $\ell = 0$ fall into the Black Hole
- If we observe this type of orbits with GRAVITY... the Galactic Center is definitly a Boson Star !