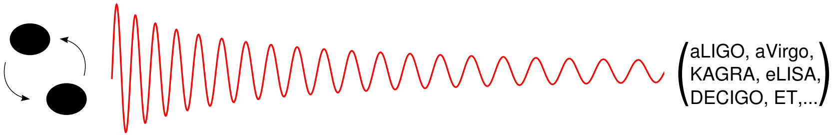
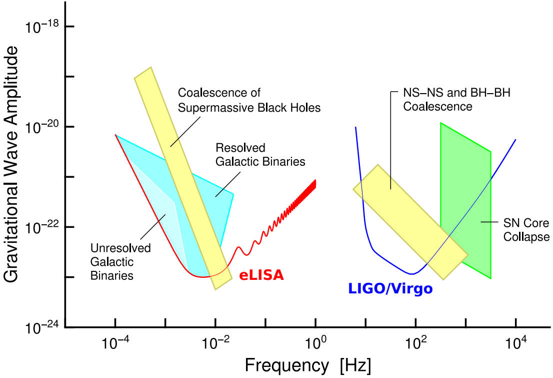
Gravitational waves from compact object binaries

Alexandre Le Tiec

Laboratoire Univers et Théories
Observatoire de Paris / CNRS

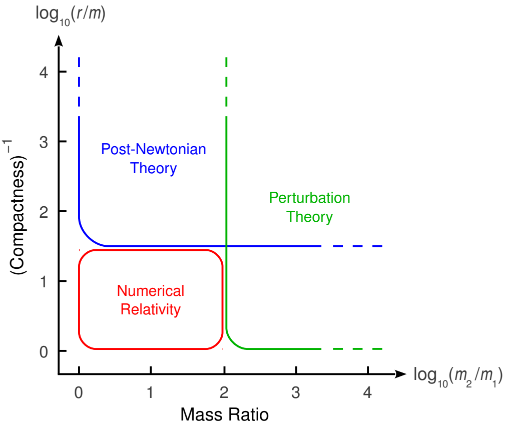


Main sources of gravitational waves (GW)



- Binary neutron stars ($2\times \sim 1.4 M_\odot$)
- Stellar mass black hole binaries ($2\times \sim 10 M_\odot$)
- Supermassive black hole binaries ($2\times \sim 10^6 M_\odot$)
- Extreme mass ratio inspirals ($\sim 10 M_\odot + \sim 10^6 M_\odot$)

Methods to compute GW templates for compact binaries



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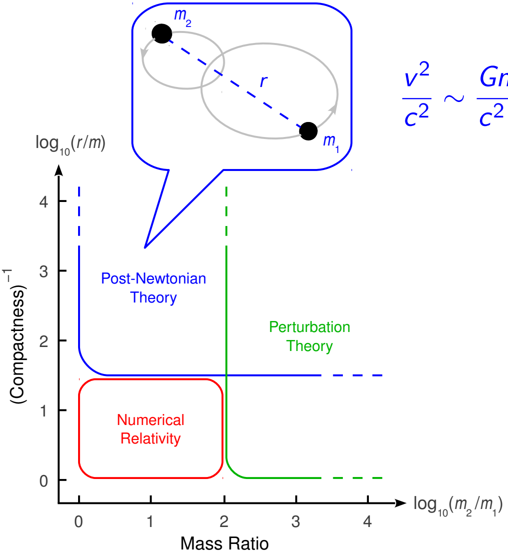
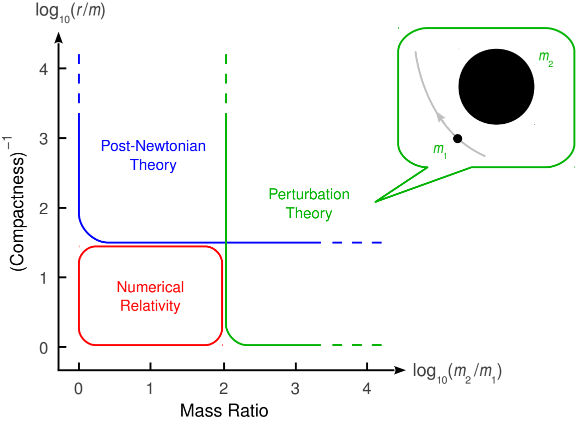


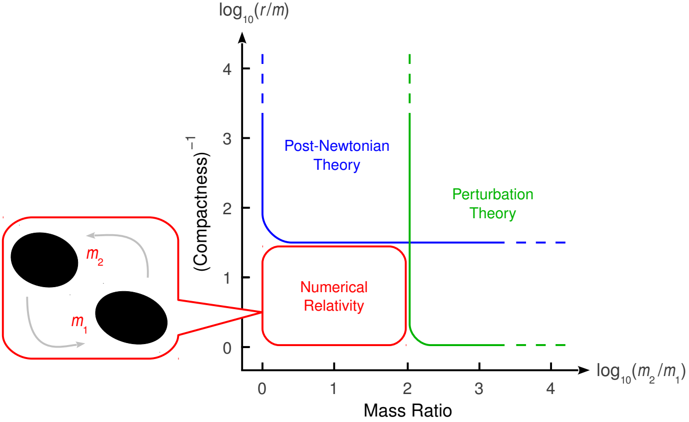
$$\frac{v^2}{c^2} \sim \frac{Gm}{c^2 r} \ll 1$$

Methods to compute GW templates for compact binaries

$$q \equiv \frac{m_1}{m_2} \ll 1$$



Methods to compute GW templates for compact binaries



Comparing the predictions from these various methods

Why?

- Independent checks of long and complicated calculations
- Identify domains of validity of approximation schemes
- Extract information inaccessible to other methods
- Develop a universal model for compact binaries

What?

- Periastron advance K *vs* orbital frequency Ω
- Binding energy E *vs* angular momentum J

Comparing the predictions from these various methods

Paper	Year	Methods	Observable	Orbit	Spin
Baker *et al.*	2007	NR/PN	waveform		
Boyle *et al.*	2007	NR/PN	waveform		
Hannam *et al.*	2007	NR/PN	waveform		
Boyle *et al.*	2008	NR/PN/EOB	energy flux		
Damour & Nagar	2008	NR/EOB	waveform		
Hannam *et al.*	2008	NR/PN	waveform		✓
Pan *et al.*	2008	NR/PN/EOB	waveform		
Campanelli *et al.*	2009	NR/PN	waveform		✓
Hannam *et al.*	2010	NR/PN	waveform		✓
Hinder *et al.*	2010	NR/PN	waveform	eccentric	
Lousto *et al.*	2010	NR/BHP	waveform		
Sperhake *et al.*	2011	NR/PN	waveform		
Sperhake *et al.*	2011	NR/BHP	waveform	head-on	
Lousto & Zlochower	2011	NR/BHP	waveform		
Nakano *et al.*	2011	NR/BHP	waveform		
Lousto & Zlochower	2013	NR/PN	waveform		
Nagar	2013	NR/BHP	recoil velocity		
Hinder *et al.*	2014	NR/PN/EOB	waveform		✓

Comparing the predictions from these various methods

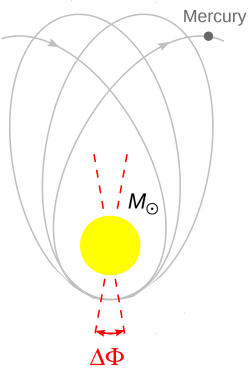
Paper	Year	Methods	Observable	Spin
Detweiler	2008	BHP/PN	redshift observable	
Blanchet *et al.*	2010	BHP/PN	redshift observable	
Damour	2010	BHP/EOB	ISCO frequency	
Mroué *et al.*	2010	NR/PN	periastron advance	
Barack *et al.*	2010	BHP/EOB	periastron advance	
Favata	2011	BHP/PN/EOB	ISCO frequency	
Le Tiec *et al.*	2011	NR/BHP/PN/EOB	periastron advance	
Damour *et al.*	2012	NR/EOB	binding energy	
Le Tiec *et al.*	2012	NR/BHP/PN/EOB	binding energy	
Akcay *et al.*	2012	BHP/EOB	redshift observable	
Hinderer *et al.*	2013	NR/EOB	periastron advance	✓
Le Tiec *et al.*	2013	NR/BHP/PN	periastron advance	✓
Bini & Damour Shah *et al.* Blanchet *et al.*	2014	BHP/PN	redshift observable	
Dolan *et al.* Bini & Damour	2014	BHP/PN	precession angle	✓
Isoyama *et al.*	2014	BHP/PN/EOB	ISCO frequency	✓

Relativistic perihelion advance of Mercury

- Observed anomalous advance of Mercury's perihelion of $\sim 43''/\text{cent.}$
- Accounted for by the leading-order relativistic angular advance per orbit

$$\Delta\Phi = \frac{6\pi G M_\odot}{c^2 a\,(1-e^2)}$$

- Periastron advance of $\sim 4^\circ/\text{yr}$ now measured in binary pulsars



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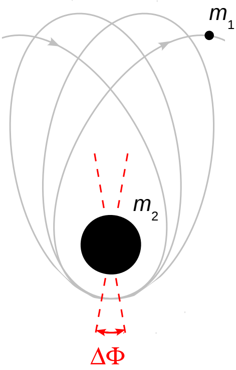
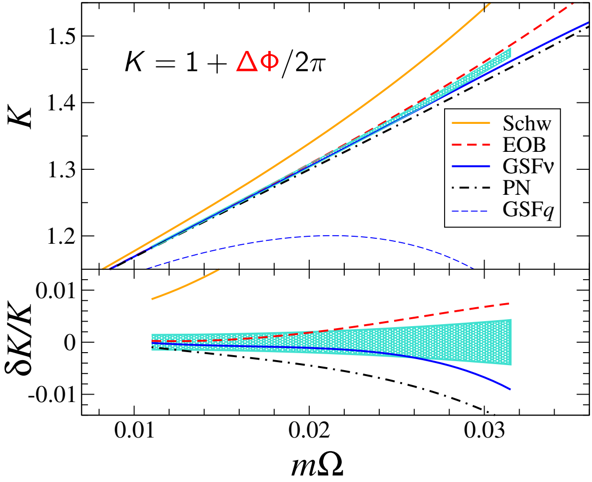


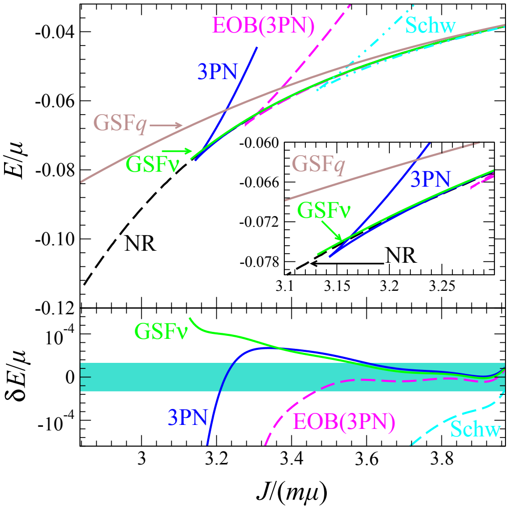
Periastron advance *vs* orbital frequency

[Le Tiec, Mroué *et al.* (2011)]

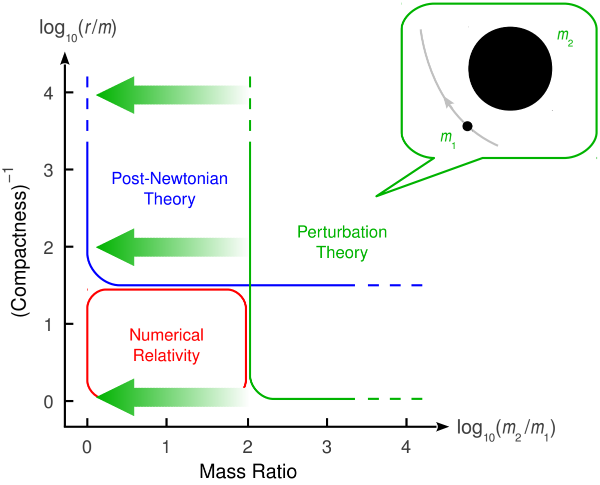


Binding energy *vs* angular momentum

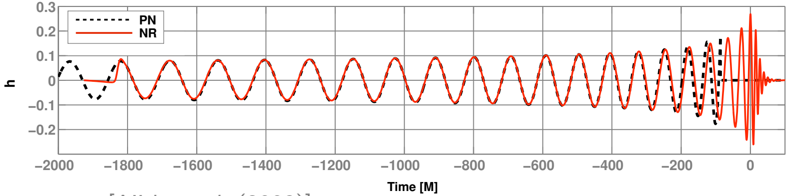
[Le Tiec, Barausse & Buonanno (2012)]



Perturbation theory for comparable-mass binaries

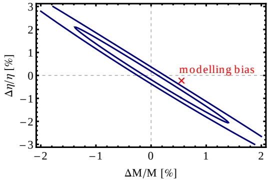


Main shortcoming of current template waveforms



[Ajith *et al.* (2008)]

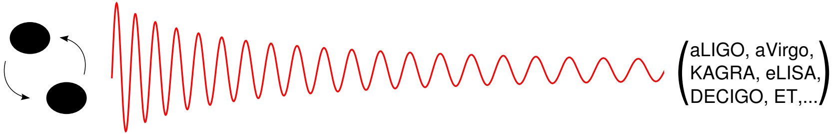
- PN breaks down during late inspiral
- Modelling error > statistical error
- Bias on parameter estimation
- Science limited by templates!



[Ohme (2012)]

Summary and prospects

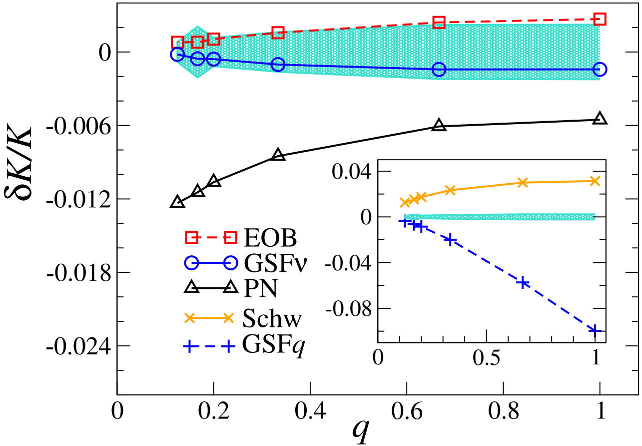
- Observing GWs will open a new window on the Universe
- Highly accurate template waveforms are a prerequisite for doing science with GW observations
- It is crucial to compare the predictions from PN theory, perturbation theory and numerical relativity
- Perturbation theory may prove useful to build templates for IMRIs and even comparable-mass binaries



Additional Material

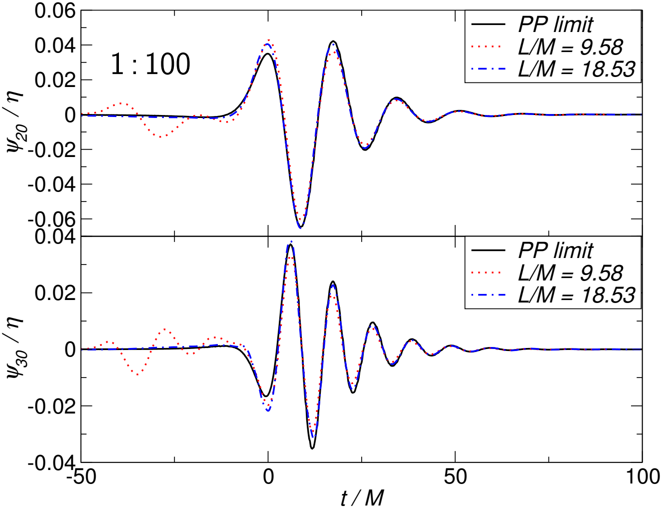
Periastron advance *vs* mass ratio

[Le Tiec, Mroué *et al.* (2011)]



Gravitational waveform for head-on collisions

[Sperhake, Cardoso *et al.* (2011)]



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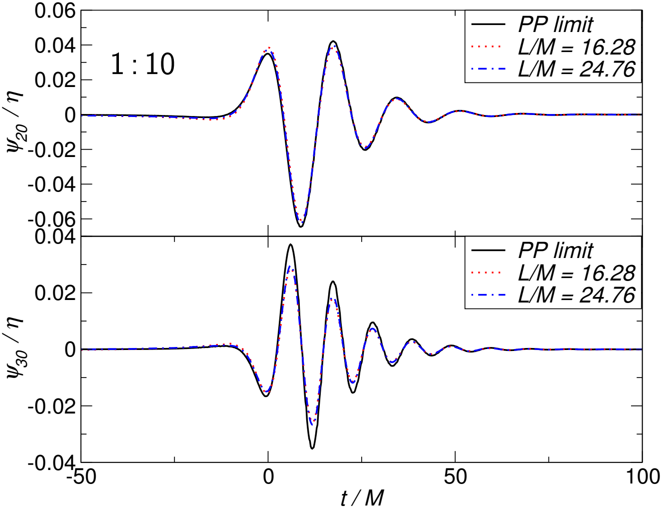
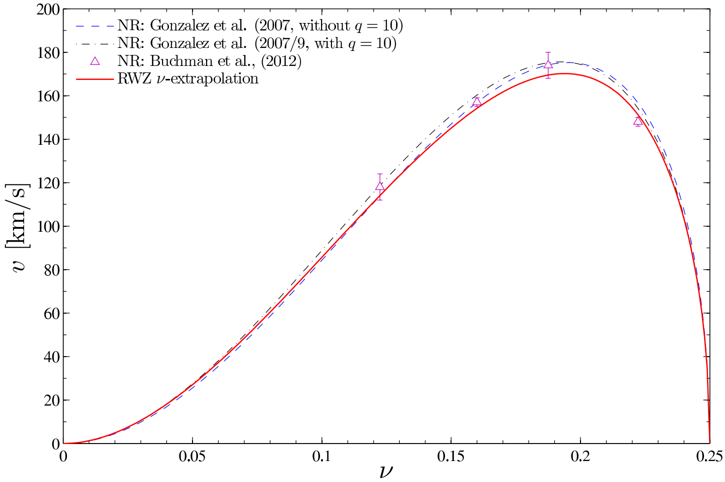


Recoil velocity for non-spinning binaries

[Nagar (2013)]



Why does perturbation theory perform so well?

- In perturbation theory, one traditionally expands as

$$\sum_{k=0}^{k_{\rm max}} A_k(m_2\Omega)\, q^k \quad \text{where} \quad q \equiv m_1/m_2 \in [0,1]$$

- However, the relations $E(\Omega; m_a)$, $J(\Omega; m_a)$, $K(\Omega; m_a)$ must be symmetric under exchange $m_1 \longleftrightarrow m_2$
- Hence, a better-motivated expansion is

$$\sum_{k=0}^{k_{\rm max}} B_k(m\Omega)\, \nu^k \quad \text{where} \quad \nu \equiv m_1 m_2/m^2 \in [0,1/4]$$

- In a PN expansion, we have $B_n = \mathcal{O}\big(1/c^{2n}\big) = n\text{PN} + \cdots$